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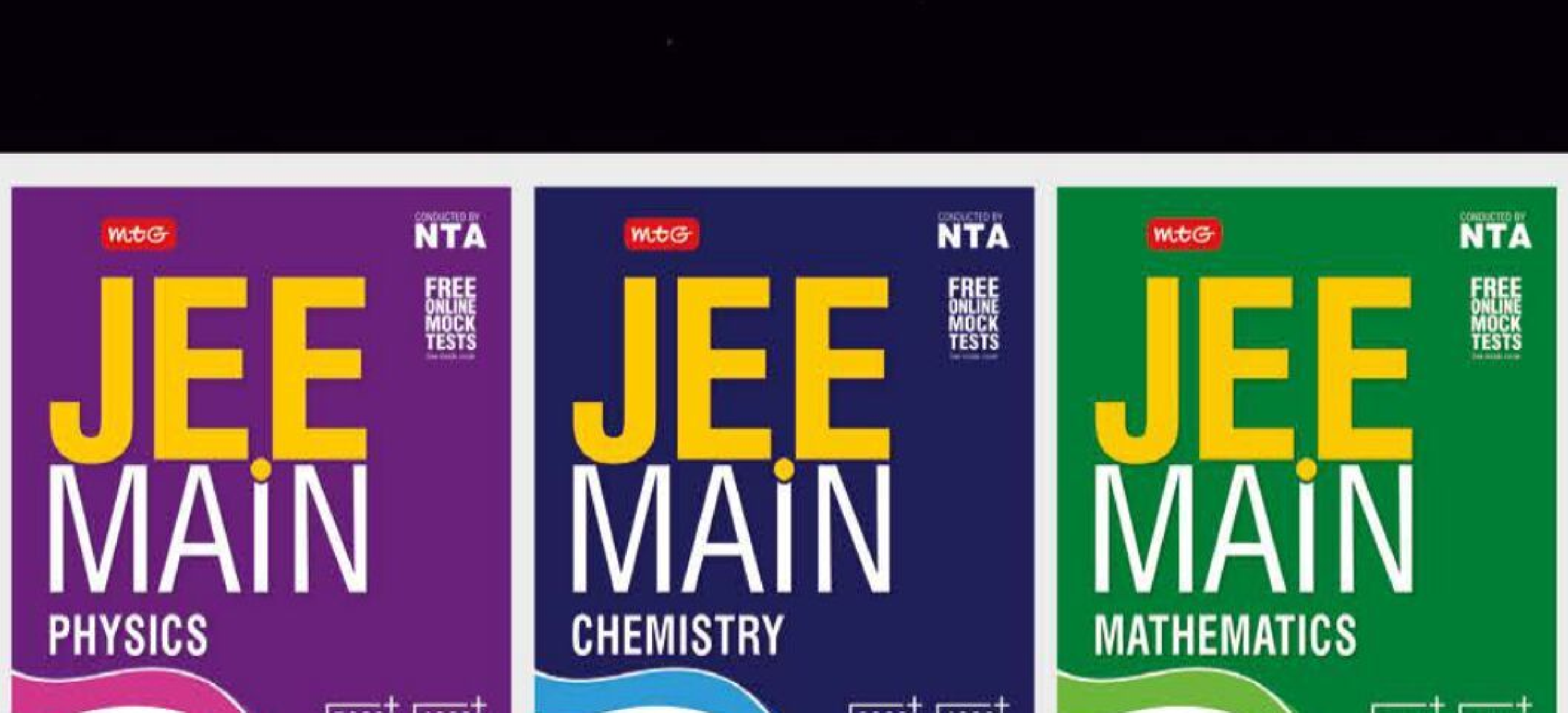
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GEAR UP for JEE MAIN 2021

Exam Dates 2021
26th, 27th, 31st August and 1st, 2nd September

Section A will be of Multiple Choice Questions (MCQs). Section B will contain questions whose answers are to be filled in as a Numerical Value. In Section B candidates have to attempt any five questions out of 10.

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The sum of the series $2\cdot{}^{20}C_0 + 5\cdot{}^{20}C_1 + 8\cdot{}^{20}C_2 + 11\cdot{}^{20}C_3 + \ldots + 62\cdot{}^{20}C_{20}$ is equal to

(a) 2^{25} (b) 2^{24} (c) 2^{26} (d) 2^{23}

2. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point $(1, 1, 0)$ is

(a) $x - 3y - 2z = -2$ (b) $2x - z = 2$
(c) $x - y - z = 0$ (d) $x + 3y + z = 4$

3. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$, $\beta = \tan^{-1}\left(\frac{1}{3}\right)$, where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to

(a) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (b) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$
(c) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (d) $\tan^{-1}\left(\frac{9}{14}\right)$

4. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{6}$ (c) $3\sqrt{6}$ (d) $\sqrt{\frac{3}{2}}$

5. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then

(a) $S_1 = \{-2, 1\}$; $S_2 = \{0\}$
(b) $S_1 = \{-1\}$; $S_2 = \{0, 2\}$
(c) $S_1 = \{-2, 0\}$; $S_2 = \{1\}$
(d) $S_1 = \{-2\}$; $S_2 = \{0, 1\}$

6. If $\cos(\alpha+\beta) = \frac{3}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$ then $\tan(2\alpha)$ is equal to

(a) $\frac{63}{52}$ (b) $\frac{21}{16}$ (c) $\frac{63}{16}$ (d) $\frac{33}{52}$

7. The sum of the coefficients of all even degree terms in x in the expansion of $(x+\sqrt{x^3-1})^6 + (x-\sqrt{x^3-1})^6, (x > 1)$ is equal to

(a) 26 (b) 32 (c) 29 (d) 24

8. The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, $(x > 0)$ is equal to

(a) 4 (b) 12 (c) 10 (d) 9

9. $\displaystyle\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}\,dx$ is equal to

(where c is a constant of integration.)

(a) $2x + \sin x + 2\sin 2x + c$
(b) $x + 2\sin x + \sin 2x + c$
(c) $x + 2\sin x + 2\sin 2x + c$
(d) $2x + \sin x + \sin 2x + c$

10. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is

(a) $\frac{7}{8}$ (b) 2 (c) $\frac{7}{4\sqrt{2}}$ (d) $\frac{11}{4\sqrt{2}}$

11. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if

(a) $r \geq 5\sqrt{\frac{3}{2}}$ (b) $\sqrt{\frac{3}{2}} < r \leq 3\sqrt{\frac{3}{2}}$
(c) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (d) $0 < r \leq \sqrt{\frac{3}{2}}$

12. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is

(a) 10 (b) 8 (c) 5 (d) 6

13. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals

(a) $2f_1(x)f_1(y)$ (b) $2f_1(x + y)f_2(x - y)$
(c) $2f_1(x)f_2(y)$ (d) $2f_1(x + y)f_1(x - y)$

14. Let $f : \mathbf{R} \to \mathbf{R}$ be a differentiable function satisfying $f'(3) + f'(2) = 0$. Then $\lim\limits_{x \to 0}\left(\dfrac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{1/x}$ is equal to

(a) 1 (b) e^{-1} (c) e (d) e^2

15. If $\displaystyle\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$

where C is a constant of integration, then the function $f(x)$ is equal to

(a) $-\dfrac{1}{2x^3}$ (b) $\dfrac{3}{x^2}$ (c) $-\dfrac{1}{6x^3}$ (d) $-\dfrac{1}{2x^2}$

16. A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is

(a) $\dfrac{100}{\sqrt{3}}$ (b) $\dfrac{100}{3}$ (c) $\dfrac{10}{\sqrt{3}}$ (d) $\dfrac{10}{3}$

17. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(a) d, e, f, are in A.P. (b) $\dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in G.P.

(c) d, e, f are in G.P. (d) $\dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in A.P.

18. Suppose that the points (h, k), $(1, 2)$ and $(-3, 4)$ lie on the line L_1. If a line L_2 passing through the points (h, k) and $(4, 3)$ is perpendicular to L_1, then k/h equals

(a) 1/3 (b) 0 (c) 3 (d) –1/7

19. If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

(a) 12 (b) 9 (c) 15 (d) 33

20. If a point $R(4, y, z)$ lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$, then the distance of R from the origin is

(a) 6 (b) $2\sqrt{21}$ (c) $\sqrt{53}$ (d) $2\sqrt{14}$

SECTION-B (NUMERICAL VALUE TYPE)

21. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is $\lambda/2$ sq. units, then value of λ is _______.

22. Consider three boxes, each containing 10 balls labelled 1, 2, ..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, $(i = 1, 2, 3)$. Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is _______.

23. Let $S_k = \dfrac{1+2+3+...+k}{k}$. If $S_1^2 + S_2^2 + ... + S_{10}^2 = \dfrac{5}{12}A$, then A is equal to _______.

24. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is _______.

25. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\dfrac{q_{21} + q_{31}}{q_{32}}$ is equal to _______.

26. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is _______.

27. If the sum of the first 15 terms of the series

$$\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$$

is equal to 225 k, then k is equal to _______.

28. The total number of irrational terms in the binomial expansion of $(7^{1/5} - 3^{1/10})^{60}$ is _______.

29. If the function f given by $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\dfrac{f(x)-14}{(x-1)^2} = 0$ $(x \neq 1)$ is _______.

30. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in metres) from the surface of the lake is _______.

SOLUTIONS

1. **(a) :** $2\cdot{}^{20}C_0 + 5\cdot{}^{20}C_1 + 8\cdot{}^{20}C_2 + 11\cdot{}^{20}C_3 + \ldots + 62\cdot{}^{20}C_{20}$

$$= \sum_{r=0}^{20}(3r+2)\,{}^{20}C_r = 3\sum_{r=0}^{20} r\,{}^{20}C_r + 2\sum_{r=0}^{20}{}^{20}C_r$$

$$= 3\sum_{r=1}^{20} r\cdot\frac{20}{r}\,{}^{19}C_{r-1} + 2\cdot 2^{20} = 3\times 20\sum_{r=1}^{20}{}^{19}C_{r-1} + 2\cdot 2^{20}$$

$= 60\cdot 2^{19} + 2\cdot 2^{20} = 2^{20}(30+2) = 2^{20}\cdot 2^{5} = 2^{25}$

2. **(c) :** The required plane is

$2x - y - 4 + \lambda(y + 2z - 4) = 0$ ...(i)

It passes through (1, 1, 0).

$\therefore\ 2 - 1 - 4 + \lambda(1 + 0 - 4) = 0$

$\Rightarrow\ -3 + \lambda(-3) = 0 \Rightarrow \lambda = -1$

Thus, the equation of plane is $2x - 2y - 2z = 0$

$\Rightarrow\ x - y - z = 0.$

3. **(b) :** Here $\alpha = \cos^{-1}\left(\frac{3}{5}\right)$ and $\beta = \tan^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow\ \cos\alpha = \frac{3}{5}$ and $\tan\beta = \frac{1}{3}$

$\Rightarrow\ \sin\alpha = \frac{4}{5},\ \sin\beta = \frac{1}{\sqrt{10}},\ \cos\beta = \frac{3}{\sqrt{10}}$

Now, $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$= \frac{4}{5}\times\frac{3}{\sqrt{10}} - \frac{3}{5}\times\frac{1}{\sqrt{10}} = \frac{9}{5\sqrt{10}}$$

$$\Rightarrow\ \alpha - \beta = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$$

4. **(d) :** The vector perpendicular to the plane containing the vector $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ is parallel to vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}(3-2) - \hat{j}(3-1) + \hat{k}(2-1) = \hat{i} - 2\hat{j} + \hat{k}$$

$\therefore$ Magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$ is given by

$$\frac{|(2\hat{i} + 3\hat{j} + \hat{k})\cdot(\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + \hat{k}|}$$

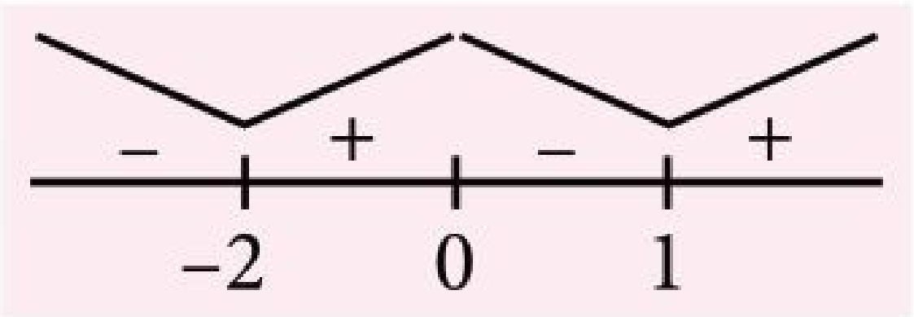
$$= \frac{|2-6+1|}{|\sqrt{1+4+1}|} = \frac{|-3|}{|\sqrt{6}|} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

5. **(a) :** Here $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

Now, $f'(x) = 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2) = 36x(x - 1)(x + 2)$

Sign of $f'(x)$



Since $f'(x)$ changes its sign from –ve to +ve about $x = -2, 1$. So, $x = -2, 1$ are the points of local minima.

$\therefore\ S_1 = \{-2, 1\}$.

Since $f'(x)$ changes its sign from +ve to –ve about $x = 0$ only. So, $x = 0$ is a point of local maxima.

$\therefore\ S_2 = \{0\}$

6. **(c) :** Given, $\cos(\alpha + \beta) = \frac{3}{5} \Rightarrow \tan(\alpha + \beta) = \frac{4}{3}$

Also, $\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$

$\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)]$

$$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}\times\frac{5}{12}} = \frac{\frac{16+5}{12}}{1 - \frac{5}{9}} = \frac{21}{12}\times\frac{9}{4} = \frac{63}{16}$$

7. **(d) :** Even degree terms of given expression

$$= \left[{}^6C_0x^6 + {}^6C_2x^4\left(\sqrt{(x^3-1)}\right)^2 + {}^6C_4x^2\left(\sqrt{x^3-1}\right)^4 + {}^6C_6x^0\left(\sqrt{x^3-1}\right)^6\right] + \left[{}^6C_0x^6 + {}^6C_2x^4(-\sqrt{x^3-1})^2 + {}^6C_4x^2\left(-\sqrt{x^3-1}\right)^4 + {}^6C_6x^0\left(-\sqrt{x^3-1}\right)^6\right]$$

$$= 2[{}^6C_0x^6 + {}^6C_2(x^7 - x^4) + {}^6C_4(x^8 + x^2 - 2x^5) + {}^6C_6(x^9 - 1 - 3x^6 + 3x^3)]$$

$\therefore$ Required sum of coefficients

$= 2[{}^6C_0 - {}^6C_2 + 2\times{}^6C_4 - {}^6C_6 - 3\times{}^6C_6]$

$= 2[1 - 15 + 30 - 1 - 3] = 24$

8. **(c) :** We have,

$$|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0, (x > 0)$$

$$\Rightarrow\ |\sqrt{x} - 2| + x - 4\sqrt{x} + 2 = 0$$

$$\Rightarrow\ |\sqrt{x} - 2| + x - 4\sqrt{x} + 2 + 2 - 2 = 0$$

$\Rightarrow\ |\sqrt{x} - 2| + (\sqrt{x} - 2)^2 - 2 = 0$ ...(i)

Let $|\sqrt{x} - 2| = t$, then (i) becomes

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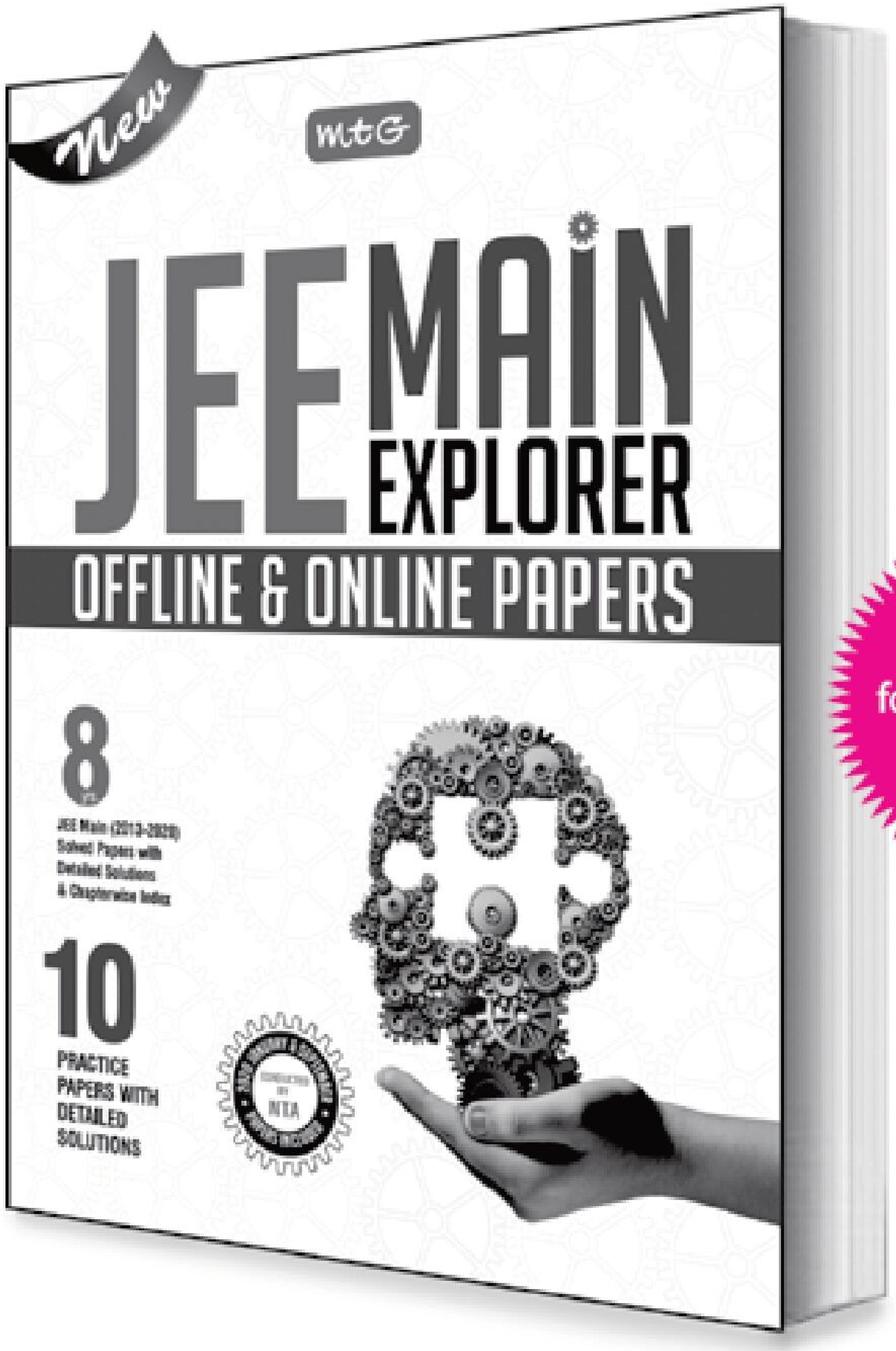
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$t^2 + t - 2 = 0 \Rightarrow \quad t^2 + 2t - t - 2 = 0$

$\Rightarrow \quad t(t + 2) - 1(t + 2) = 0 \Rightarrow (t - 1)(t + 2) = 0$

$\Rightarrow \quad t = 1, t = -2$

Since $|\sqrt{x} - 2| = -2$ not possible

So, $\sqrt{x} - 2 = \pm 1 \Rightarrow \sqrt{x} = 3, \sqrt{x} = 1$

$\Rightarrow \quad x = 9, x = 1$

So, required sum = 9 + 1 = 10

9. (b) : Let $I = \int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\frac{x}{2}} dx = \int \frac{2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} dx$

$$= \int \frac{\sin\left(\frac{5x}{2}+\frac{x}{2}\right)+\sin\left(\frac{5x}{2}-\frac{x}{2}\right)}{\sin x} dx$$

$$= \int \frac{\sin(3x)+\sin 2x}{\sin x} dx$$

$$= \int \frac{3\sin x - 4\sin^3 x + 2\sin x\cos x}{\sin x} dx$$

$$= \int (3 - 4\sin^2 x + 2\cos x) dx$$

$$= \int \left(3 - 4\left(\frac{1-\cos 2x}{2}\right) + 2\cos x\right) dx$$

$$= \int (3 - 2 + 2\cos 2x + 2\cos x) dx$$

$$= x + 2\frac{\sin 2x}{2} + 2\sin x + c = x + \sin 2x + 2\sin x + c$$

10. (c) : We are given the line

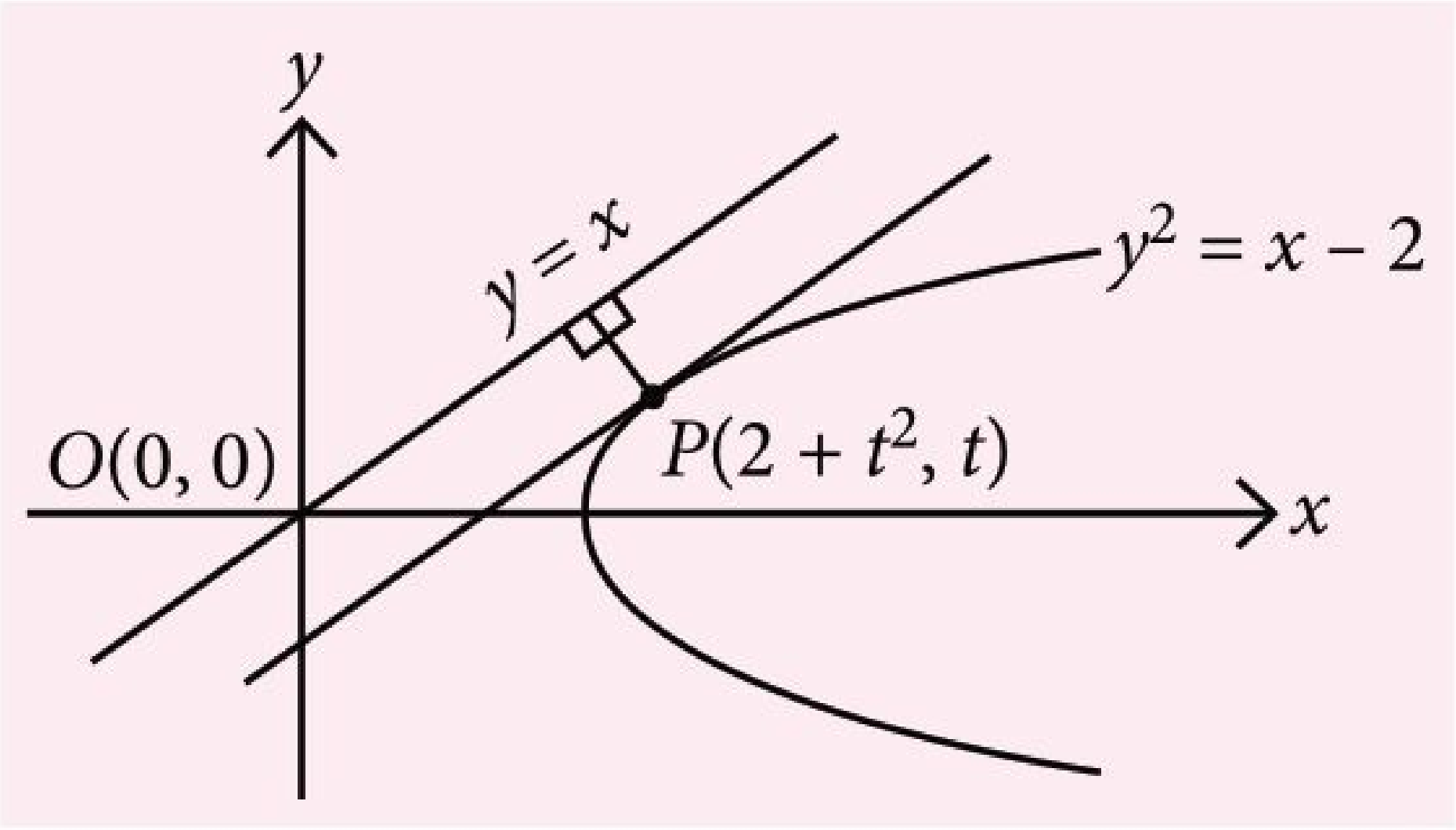
$y = x \Rightarrow \frac{dy}{dx} = 1$

and the curve

$y^2 = x - 2$

$\Rightarrow \quad 2y\frac{dy}{dx} = 1$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2y}$



Let $P(t^2 + 2, t)$ be any point on the curve.

Thus, $\left(\frac{dy}{dx}\right)_{(t^2+2,\, t)} = \frac{1}{2t}$

Now, $\frac{1}{2t} = 1 \Rightarrow 2t = 1 \Rightarrow t = \frac{1}{2}$

So, $P \equiv \left(\frac{9}{4}, \frac{1}{2}\right)$

Hence, shortest distance $= \frac{\left|\frac{9}{4} - \frac{1}{2}\right|}{\sqrt{1^2+1^2}} = \frac{7}{4\sqrt{2}}$ units

11. (a) : Given $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}(2 + x) - \hat{j}(3 - x) + \hat{k}(-3 - 2) = (2 + x)\hat{i} + (x - 3)\hat{j} - 5\hat{k}$

Now, $|\vec{a} \times \vec{b}| = \sqrt{(2+x)^2 + (x-3)^2 + (-5)^2}$

$= \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25} = \sqrt{2x^2 - 2x + 38}$

Let $f(x) = 2x^2 - 2x + 38 \Rightarrow f'(x) = 4x - 2 = 2(2x - 1)$

Put $f'(x) = 0 \Rightarrow x = \frac{1}{2}$

Also, $f''(x) = 4 > 0 \ \forall x$

So, $f(x)$ is minimum for $x = \frac{1}{2}$.

$\therefore$ $|\vec{a} \times \vec{b}|$ is minimum for $x = \frac{1}{2}$.

Thus, $|\vec{a} \times \vec{b}| \geq \sqrt{2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 38} = \sqrt{\frac{75}{2}} = 5\sqrt{\frac{3}{2}}$

12. (c) : Let equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$

Now, $2a - 2b = 10 \Rightarrow a - b = 5$ ...(i)

Also, $ae = 5\sqrt{3}$ ...(ii)

Now, $a^2e^2 = a^2 - b^2 \Rightarrow (5\sqrt{3})^2 = a^2 - b^2$ (Using (ii))

$\Rightarrow \quad (a + b)(a - b) = 75$

$\Rightarrow \quad a + b = 15$ ...(iii) (Using (i))

Solving (i) and (iii), we get

$a = 10, b = 5$

$\therefore$ Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times (5)^2}{10} = \frac{50}{10} = 5$

13. (a) : Given, $f(x) = a^x$, $a > 0$

$\Rightarrow \quad f(x) = \frac{2a^x}{2} = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$

$\Rightarrow \quad f_1(x) = \frac{a^x + a^{-x}}{2}$ and $f_2(x) = \frac{a^x - a^{-x}}{2}$

Consider $f_1(x + y) + f_1(x - y)$

$$=\frac{a^{x+y}+a^{-(x+y)}}{2}+\frac{a^{x-y}+a^{-(x-y)}}{2}$$

$$=\frac{a^{x+y}}{2}+\frac{a^{-x+y}}{2}+\frac{a^{-x-y}}{2}+\frac{a^{x-y}}{2}$$

$$=a^{y}\left(\frac{a^{x}+a^{-x}}{2}\right)+a^{-y}\left(\frac{a^{x}+a^{-x}}{2}\right)$$

$$=\left(\frac{a^{x}+a^{-x}}{2}\right)(a^{y}+a^{-y}) = f_1(x)\times 2f_1(y) = 2f_1(x)f_1(y)$$

14. (a) : Consider,

$$\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}} \qquad (1^{\infty}\text{ form})$$

$$=\lim_{x\to 0}\left(1+\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$$

$$=e^{\lim\limits_{x\to 0}\frac{f(3+x)-f(3)-f(2-x)+f(2)}{x[1+f(2-x)-f(2)]}}$$

$$=e^{\lim\limits_{x\to 0}\frac{f(3+x)-f(3)}{x\cdot 1}}\cdot e^{\lim\limits_{x\to 0}\frac{f(2)-f(2-x)}{x\cdot 1}}$$

$$=e^{f'(3)}.e^{f'(2)}=e^{f'(3)+f'(2)}$$

$$=e^{0} \qquad [\because\ f'(3)+f'(2)=0]$$

$$=1$$

15. (a) : Given, $\int\frac{dx}{x^{3}(1+x^{6})^{2/3}} = xf(x)(1+x^{6})^{1/3}+C$

$$\Rightarrow \int\frac{dx}{x^{7}\left(\frac{1}{x^{6}}+1\right)^{2/3}}=xf(x)(1+x^{6})^{1/3}+C \qquad \text{...(i)}$$

Put $\frac{1}{x^{6}}+1=t \;\Rightarrow\; \frac{-6}{x^{7}}dx=dt \;\Rightarrow\; \frac{dx}{x^{7}}=\frac{-1}{6}dt$

$$\therefore\ \int\frac{dx}{x^{7}\left(\frac{1}{x^{6}}+1\right)^{2/3}}=\frac{-1}{6}\int\frac{dt}{t^{2/3}}=\frac{-1}{6}\times 3(t)^{1/3}+C$$

$$=\frac{-1}{2}\left(\frac{1}{x^{6}}+1\right)^{1/3}+C=\frac{-1}{2}\left(\frac{x^{6}+1}{x^{6}}\right)^{1/3}+C$$

$$=\frac{-1}{2x^{2}}(x^{6}+1)^{1/3}+C=x\left(\frac{-1}{2x^{3}}\right)(1+x^{6})^{1/3}+C \qquad \text{...(ii)}$$

Comparing (i) and (ii), we get

$$f(x)=\frac{-1}{2x^{3}}$$

16. (c) : Let the student scores x marks in the sixth test.

Now, mean $(\bar{x})=\frac{45+54+41+57+43+x}{6}$

$\Rightarrow\ 48\times 6=240+x \qquad [\because\ \text{Mean} = 48 \text{ (Given)}]$

$\Rightarrow\ x=288-240=48$

$$\text{Variance }(\sigma^{2})=\frac{\Sigma x_i^{2}}{n}-(\bar{x})^{2}$$

$$=\frac{(45)^{2}+(54)^{2}+(41)^{2}+(57)^{2}+(43)^{2}+(48)^{2}}{6}-(48)^{2}$$

$$=\frac{14024}{6}-2304=\frac{200}{6}=\frac{100}{3}$$

$\therefore$ Standard deviation $(\sigma) = 10/\sqrt{3}$

17. (d) : Given a, b, c are in G.P.

Let $a = a$, $b = ar$ and $c = ar^{2}$ ...(i)

Now, consider $ax^{2} + 2bx + c = 0$

$\therefore\ ax^{2} + 2arx + ar^{2} = 0$ [Using (i)]

$\Rightarrow\ x^{2} + 2rx + r^{2} = 0 \Rightarrow (x + r)^{2} = 0 \Rightarrow x = -r$

Since, $x = -r$ is also a root of $dx^{2} + 2ex + f = 0$

$\therefore\ dr^{2} - 2er + f = 0$

$$\Rightarrow\ d\left(\frac{c}{a}\right)-2e\left(\frac{c}{b}\right)+f=0$$

$$\left[\because\ (-r)(-r)=\frac{c}{a}\text{ and }\frac{ar^{2}}{ar}=\frac{c}{b}\right]$$

PUBLIC NOTICE

Rescheduling of Session 4 of JEE Main 2021

In view of the persistent demand from the student community and to enable the candidates to maximize their performance, the National Testing Agency has decided to provide a gap of four weeks between Session 3 and Session 4 of JEE Main 2021. Accordingly, the new dates of the JEE Main 2021 Session 4 are **26, 27, 31 August, and 1, 2 September 2021**. The total number of candidates who have already registered for the JEE Main 2021 Session 4 is 7.32 lacs.

JEE Main 2021	Date of Exams	Downloading of e-Admit Card	Declaration of Result
Session 4 B.E./B.Tech. (Paper-1) & B.Arch. (Paper-2A) / B.Planning (Paper-2B)	26, 27, 31 August, and 1, 2 September 2021	To be intimated later through JEE Main Portal	

The Candidates are advised to keep visiting the official websites of NTA ***www.nta.ac.in*** and ***jeemain.nta.nic.in*** for the latest updates.

$$\Rightarrow \quad \frac{dc}{a}+f=\frac{2ec}{b} \quad \Rightarrow \frac{d}{a}+\frac{f}{c}=\frac{2e}{b}$$

$\therefore \quad \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

18. (a) : Line L_1 passes through (1, 2) and (–3, 4).

$\therefore$ Equation of line L_1 is

$$y-4=\left(\frac{4-2}{-3-1}\right)(x+3)$$

$$\Rightarrow \quad y-4=\frac{-1}{2}(x+3) \quad \Rightarrow \quad y=\frac{-x}{2}+\frac{5}{2} \qquad \text{...(i)}$$

Now, slope of line L_2 is 2. $[\because L_2 \perp L_1]$

$\therefore$ Equation of line L_2 passing through (4, 3) is

$y - 3 = 2(x - 4) \Rightarrow y = 2x - 5$ ...(ii)

Solving (i) and (ii), we get $x = 3$ and $y = 1$

$$\Rightarrow h = 3 \text{ and } k = 1 \qquad \therefore \quad \frac{k}{h}=\frac{1}{3}$$

19. (d) : Let $y=f(f(f(x)))+(f(x))^2$

$$\Rightarrow \quad \frac{dy}{dx}=f'(f(f(x)))f'(f(x))f'(x)+2f(x)f'(x)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1}=f'(f(f(1)))f'(f(1))f'(1)+2f(1)f'(1)$$

$= f'(1)f'(1)f'(1)+2f'(1)$ $[\because f(1) = 1]$

$= (3)^3 + 2 \times 3 = 27 + 6 = 33$ $[\because f'(1) = 3]$

20. (d) : Given, $R(4, y, z)$ lies on the line segment joining the points $P(2, –3, 4)$ and $Q(8, 0, 10)$.

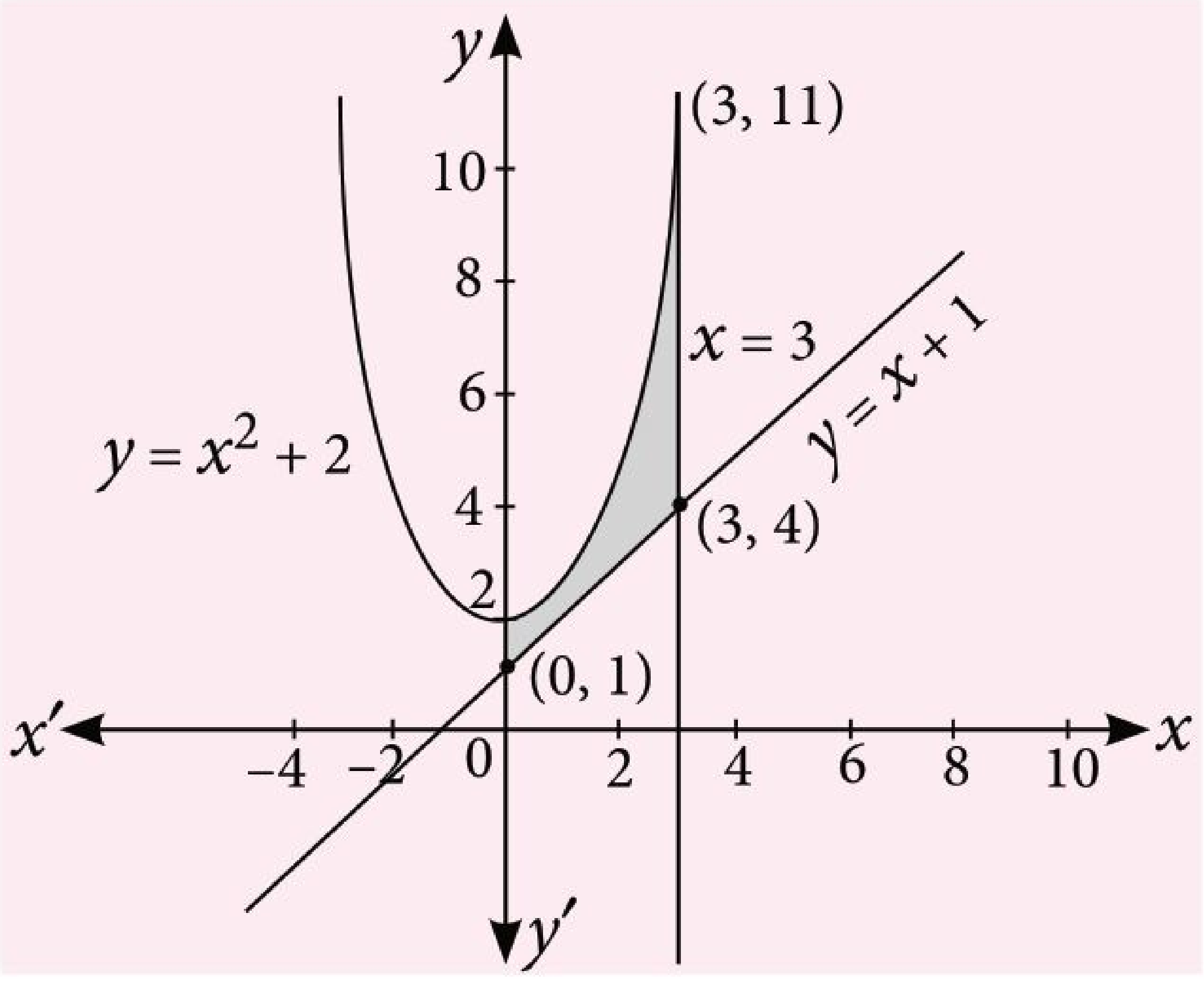
$$\therefore \quad \frac{8-4}{4-2}=\frac{0-y}{y+3}=\frac{10-z}{z-4} \quad \Rightarrow \quad \frac{4}{2}=\frac{-y}{y+3}=\frac{10-z}{z-4}$$

$\therefore$ $y = –2$ and $z = 6$

$\therefore$ Distance of R (4, –2, 6) from O (0, 0, 0)

$=\sqrt{16+4+36}=\sqrt{56}=2\sqrt{14}$ units

21. (15) : The shaded region represents the required area in the figure.



$$\text{Required area}=\int_0^3[(x^2+2)-(x+1)]dx=\int_0^3(x^2-x+1)\,dx$$

$$=\left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]_0^3=9-\frac{9}{2}+3=\frac{15}{2}\text{ sq. units}$$

$\therefore$ $\lambda = 15$

22. (120) : Let $n_1 = 1$, then n_2 can be 2, 3, ..., 9 and n_3 can be 3, ..., 10

$\therefore$ No. of ways = 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = $\frac{8\times 9}{2}$

Similarly, when $n_1 = 2$, then n_2 can be 3, ..., 9 and n_3 can be 4, ..., 10

$\therefore$ No. of ways = 7 + 6 + 5 + 4 + 3 + 2 + 1 = $\frac{7\times 8}{2}$

And so on.

$\therefore$ Required number of ways

$$=\frac{8\times 9}{2}+\frac{7\times 8}{2}+....+\frac{2\times 3}{2}+\frac{1\times 2}{2}=\frac{240}{2}=120$$

23. (303) : $S_k=\frac{k(k+1)}{2k}=\frac{k+1}{2}$

$$\text{Now, } \sum_{k=1}^{10}(S_k)^2=\frac{5}{12}A \quad \Rightarrow \quad \sum_{k=1}^{10}\left(\frac{k+1}{2}\right)^2=\frac{5}{12}A$$

$$\Rightarrow \quad \frac{1}{4}(2^2+3^2+....+11^2)=\frac{5}{12}A$$

$$\Rightarrow \quad \frac{1}{4}\left(\frac{11\times 12\times 23}{6}-1\right)=\frac{5}{12}A$$

$$\Rightarrow \quad \frac{505}{4}=\frac{5}{12}A \quad \Rightarrow \quad A = 303$$

24. (31) : Here, $\sum_{i=1}^{50}(x_i-30)=50$

$$\Rightarrow \quad \sum_{i=1}^{50}x_i=50+50\times 30$$

$$\therefore \quad \text{Mean}=\frac{\sum_{i=1}^{50}x_i}{50}=\frac{50\times 31}{50}=31$$

25. (10) : Here, $P=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$ and $Q = P^5 + I_3$

$$\therefore \quad P^2=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}=\begin{bmatrix}1&0&0\\6&1&0\\27&6&1\end{bmatrix}$$

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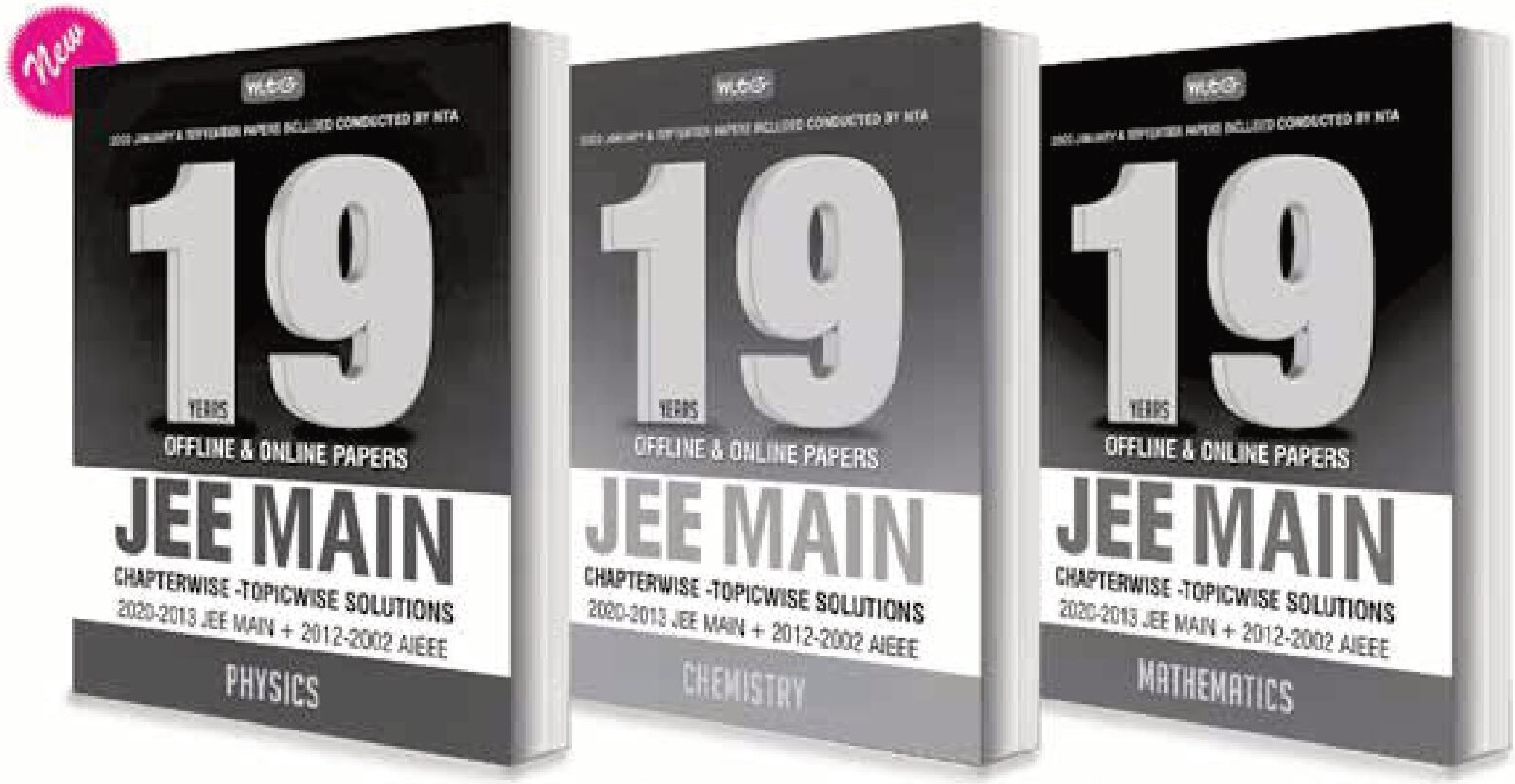
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$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 9 & 1 & 0 \\ 54 & 9 & 1 \end{bmatrix}$$

Similarly, $P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$

$$\therefore \quad Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix} \Rightarrow \frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = 10$$

26. (28) : Let 3 consecutive terms are $\frac{a}{r}$, a, ar of a G.P.

$\therefore \quad a^3 = 512 \Rightarrow a = 8$

Now, $\frac{8}{r}+4$, 12, $8r$ are in A.P.

$\therefore \quad 24 = \frac{8}{r}+4+8r \Rightarrow 24r = 8 + 4r + 8r^2$

$\Rightarrow \quad 2r^2 - 5r + 2 = 0 \Rightarrow (r - 2)(2r - 1) = 0$

$\Rightarrow \quad r = 2 \text{ or } \frac{1}{2}$

When $r = 2$, terms are 4, 8, 16

When $r = \frac{1}{2}$, terms are 16, 8, 4

So, required sum = 16 + 8 + 4 = 28

27. (27) : Let

$$S = \left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots \text{ 15 terms}$$

$$= \left(\frac{3}{4}\right)^3 + \left(\frac{3}{2}\right)^3 + \left(\frac{9}{4}\right)^3 + 3^3 + \left(\frac{15}{4}\right)^3 + \ldots \text{ 15 terms}$$

$$= \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + \left(\frac{15}{4}\right)^3 + \ldots \text{ 15 terms}$$

$$= \frac{27}{64}\sum_{x=1}^{15} x^3 = \frac{27}{64}\left[\frac{15(15+1)}{2}\right]^2 = 27 \times 225$$

$\therefore \quad 225\,k = 27 \times 225 \Rightarrow k = 27$

28. (54) : Given expression is $(7^{1/5} - 3^{1/10})^{60}$

General term of this expansion is

$$T_{r+1} = (-1)^r \, {}^{60}C_r \, 7^{\frac{60-r}{5}} \, 3^{\frac{r}{10}}$$

For rational terms, r = 0, 10, 20, 30, 40, 50, 60

So, number of rational terms = 7

$\therefore$ Number of irrational terms = 61 – 7 = 54

29. (7) : Here, $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$

$\therefore \quad f'(x) = 3x^2 - 6(a - 2)\,x + 3a$

Since $f'(x) \geq 0 \;\; \forall \; x \in (0, 1]$ and $f'(x) \leq 0 \; \forall \; x \in [1, 5)$

$\therefore \quad f'(x) = 0$ at $x = 1$

$\Rightarrow \quad 3 - 6(a - 2) + 3a = 0$

$\Rightarrow \quad -3a + 15 = 0 \Rightarrow a = 5$

So, $f(x) = x^3 - 9x^2 + 15x + 7$

Now, $\frac{f(x)-14}{(x-1)^2} = 0 \Rightarrow \frac{x^3-9x^2+15x-7}{(x-1)^2} = 0$

$\Rightarrow \quad \frac{(x-1)^2(x-7)}{(x-1)^2} = 0 \Rightarrow x = 7$

30. (50) : In ΔPAB,

$\tan 30° = \frac{x}{y} \Rightarrow y = \sqrt{3}x$ ...(i)

In ΔPBC,

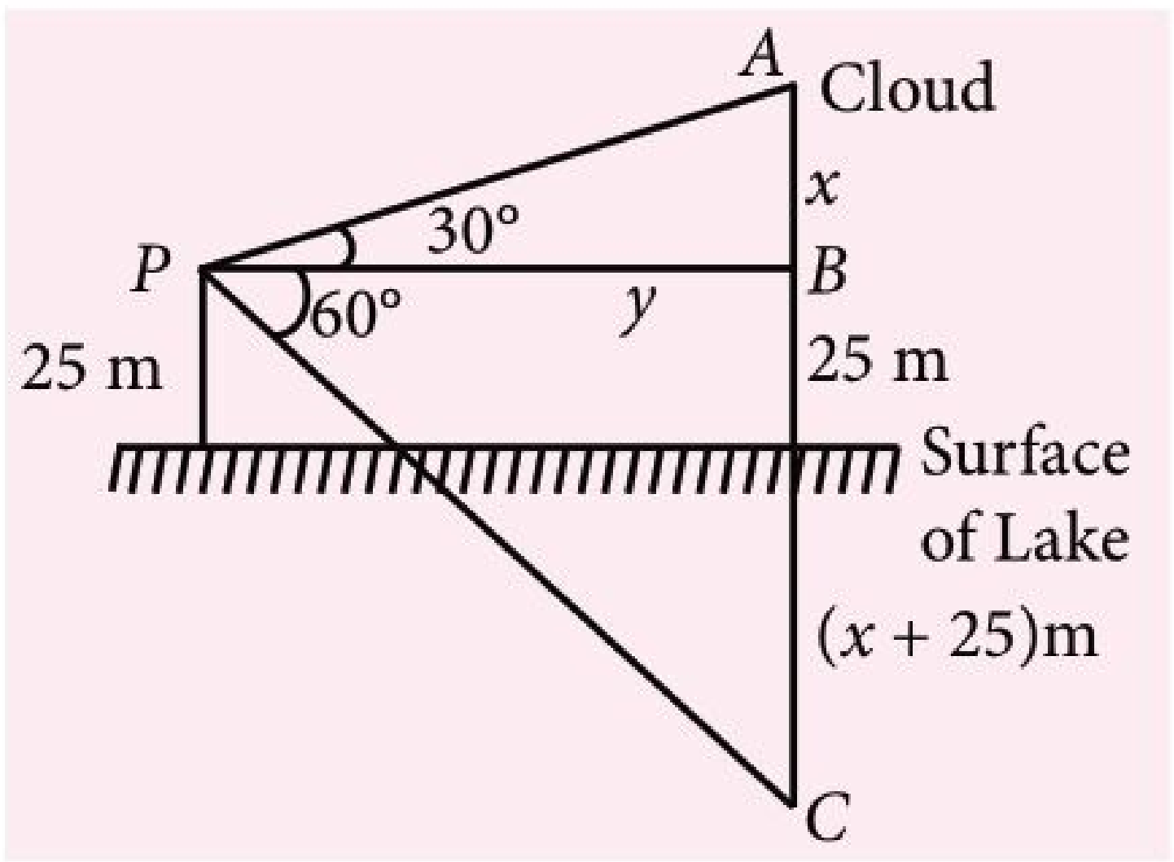
$\tan 60° = \frac{25+x+25}{y}$

$\Rightarrow \quad \sqrt{3}y = 50 + x$

$\Rightarrow \quad 3x = 50 + x \quad$ [Using (i)] $\Rightarrow \quad x = 25$

$\therefore$ Height of cloud from the surface of the lake

= 25 + 25 = 50 m



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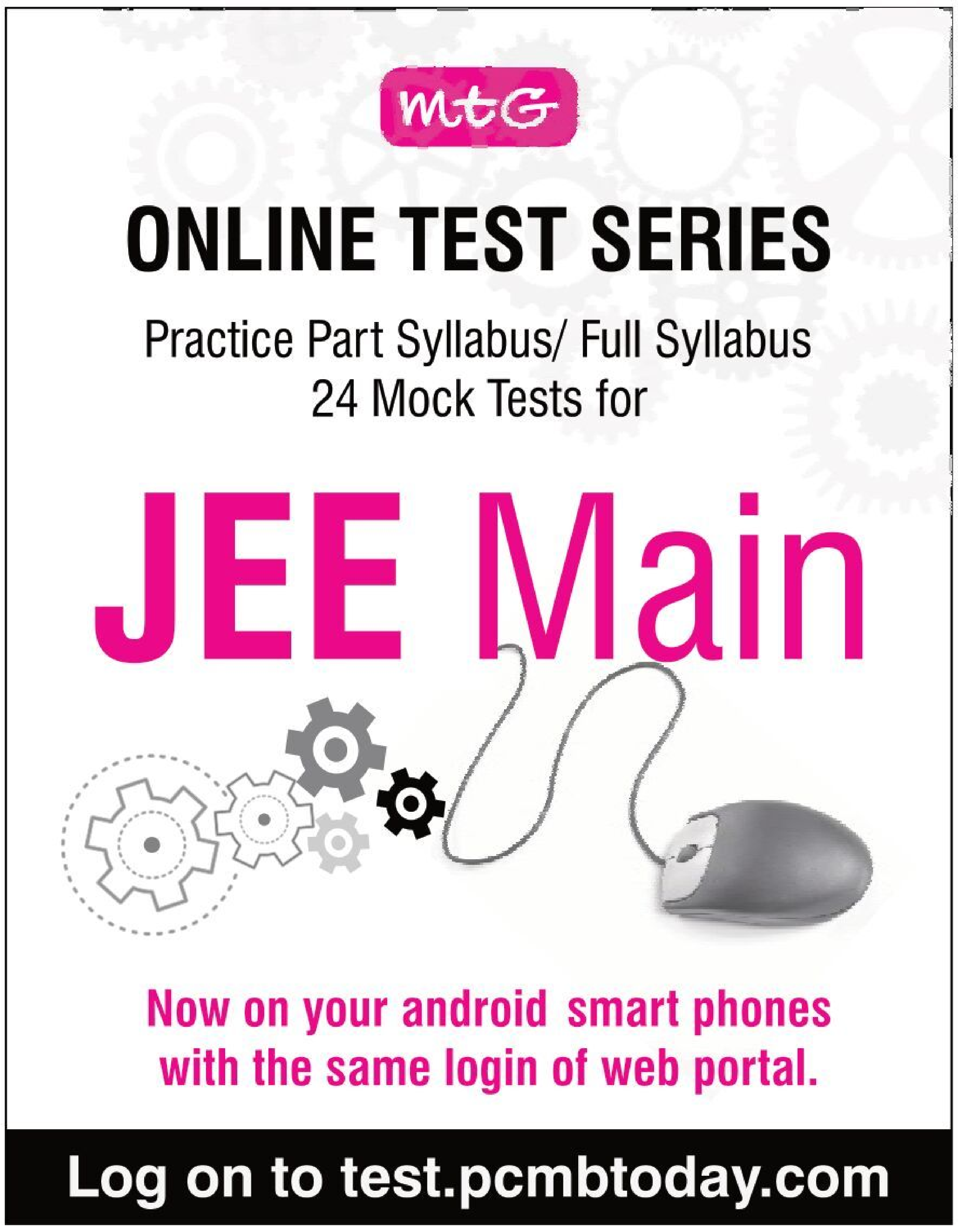
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PAPER-1

SINGLE OPTION CORRECT TYPE

1. All the roots of the equation
$11z^{10} + 10iz^9 + 10iz - 11 = 0$ lie

(a) inside $|z| = 1$ (b) on $|z| = 1$
(c) outside $|z| = 1$ (d) none of these

2. Let $y = f(x)$ be a continuous and differentiable curve. The normals at $(1, f(1))$, $(2, f(2))$ and $(3, f(3))$ make angles $\pi/3$, $\pi/4$ and $\pi/6$ with positive x-axis respectively. Then value of $\int_1^2 \left(f(x) + xf'(x)\right)dx + \int_2^3 f'(x)\,f''(x)\,dx$ is equal to

(a) $f'(2) + f(1) + 1$ (b) $2f(2) - f(1) + 1$
(c) $f(3) - f(2) + 1$ (d) none of these

3. If $I_1 = \int_{-100}^{101} \frac{dx}{(5 + 2x - 2x^2)(1 + e^{2-4x})}$ and

$I_2 = \int_{-100}^{101} \frac{dx}{5 + 2x - 2x^2}$, then $\frac{I_1}{I_2}$ is

(a) 1/2 (b) 2
(c) 0 (d) none of these

4. If $\tan A$, $\tan B$ are the roots of the quadratic equation $abx^2 - c^2x + ab = 0$ where a, b, c are sides of the ΔABC then $\sin^2 A + \sin^2 B + \sin^2 C$ is

(a) 1 (b) 2 (c) 3 (d) 3/2

5. If three one by one squares are selected at random from the chessboard, then the probability that they form the letter 'L' is

(a) $\frac{196}{^{64}C_3}$ (b) $\frac{49}{^{64}C_3}$ (c) $\frac{36}{^{64}C_3}$ (d) $\frac{98}{^{64}C_3}$

6. Let a, b, c be distinct non-negative numbers and the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$, $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then the quadratic equation $ax^2 + 2cx + b = 0$ has

(a) real and equal roots
(b) real and unequal roots
(c) both roots real and positive
(d) none of these

ONE OR MORE THAN ONE OPTION(S) CORRECT TYPE

7. Let $f(x) = \begin{cases} \int_0^x (5 + |1 - t|)\,dt, & \text{if} \quad x > 2 \\ 5x + 1, & \text{if} \quad x \le 2 \end{cases}$ then the function is

(a) continuous at $x = 2$
(b) differentiable at $x = 2$
(c) discontinuous at $x = 2$
(d) not differentiable at $x = 2$

8. The plane $lx + my = 0$ is rotated about its line of intersection with the plane $z = 0$, through an angle α, then equation of plane in its new position may be

(a) $lx + my + z\sqrt{l^2 + m^2}\,\tan\alpha = 0$
(b) $lx + my - z\sqrt{l^2 + m^2}\,\tan\alpha = 0$
(c) data is not sufficient
(d) none of these

9. Let a, b, c be distinct complex numbers with $|a| = |b| = |c| = 1$ and z_1, z_2 be the roots of the equation $az^2 + bz + c = 0$ with $|z| = 1$. Let P and Q represent the complex numbers z_1 and z_2 in the argand plane with $\angle POQ = \theta$, $0° < \theta < 180°$ (where O being the origin), then

(a) $b^2 = ac;\ \theta = \frac{2\pi}{3}$ (b) $\theta = \frac{2\pi}{3};\ PQ = \sqrt{3}$
(c) $PQ = 2\sqrt{3};\ b^2 = ac$ (d) $\theta = \frac{\pi}{3};\ b^2 = ac$

10. If $f_n(\alpha) = \tan\frac{\alpha}{2}(1 + \sec\alpha)(1 + \sec 2\alpha)(1 + \sec 4\alpha)$ .....$(1 + \sec 2^n\alpha)$, $n \in N$, then

(a) $f_2\left(\frac{\pi}{16}\right)=f_5\left(\frac{\pi}{128}\right)$

(b) $f_3\left(\frac{\pi}{32}\right)=f_4\left(\frac{\pi}{64}\right)$

(c) $f_2\left(\frac{\pi}{16}\right)=f_4\left(\frac{\pi}{64}\right)$

(d) $f_2\left(\frac{\pi}{16}\right)=f_3\left(\frac{\pi}{32}\right)=f_4\left(\frac{\pi}{64}\right)=f_5\left(\frac{\pi}{128}\right)=1$

11. If Rolle's theorem is applicable to $f(x)=\frac{\log x}{x}$ over the interval $[a, b]$, $(a, b \in I^+)$ then possible values of a and b are

(a) $a = 2$ (b) $a = 3$

(c) $b = 4$ (d) $b = 5$

12. If a variable point P on an ellipse of eccentricity (e) is joined to foci S_1 and S_2, then locus of incentre of ΔPS_1S_2 is

(a) an ellipse

(b) a hyperbola

(c) a conic with eccentricity $\sqrt{\frac{2e}{1+e}}$

(d) a conic with eccentricity $\sqrt{\frac{2}{1+e}}$

NUMERICAL VALUE TYPE

13. An ellipse has foci at (9, 20) and (49, 55) in the xy-plane and is tangent to x-axis. The length of its major axis is ______.

14. Let $S(n)$ denotes sum of first n terms of an A.P., then the value of $S=\lim_{n\to\infty}\sum_{r=-n}^{n}\frac{f(r)}{n}$, where $f(n)=\frac{S(3n)}{S(2n)-S(n)}$, is ______.

15. If $\lim_{x\to 0}\left(1+x+\frac{f(x)}{x}\right)^{1/x}=e^3$, then $\lim_{x\to 0}\frac{f(x)}{x^2}=$ ______.

16. The sum of all the coefficients of those terms in the expansion of $(a + b + c + d)^8$ which contain b but not c is ______.

17. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots\ldots + 20(21)^{19} = m(20)^{19}$, then the value of $\sqrt{m+41}$ is ____.

18. If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ and $\vec{\delta}$ are unit vectors, then

$$|\vec{\alpha}-\vec{\beta}|^2+|\vec{\beta}-\vec{\gamma}|^2+|\vec{\gamma}-\vec{\delta}|^2+|\vec{\delta}-\vec{\alpha}|^2+|\vec{\gamma}-\vec{\alpha}|^2+|\vec{\beta}-\vec{\delta}|^2$$

does not exceed ______.

PAPER-2

SINGLE DIGIT INTEGER ANSWER TYPE

1. Let α, β, γ be distinct real numbers such that

$a\alpha^2 + b\alpha + c = (\sin\theta)\,\alpha^2 + (\cos\theta)\alpha$

$a\beta^2 + b\beta + c = (\sin\theta)\,\beta^2 + (\cos\theta)\beta$

$a\gamma^2 + b\gamma + c = (\sin\theta)\,\gamma^2 + (\cos\theta)\gamma$

(where $a, b, c \in R$)

Then the maximum value of the expression $\frac{a^2+b^2}{a^2+3ab+5b^2}$ is ______.

2. In a sequence of circles $C_1, C_2, C_3, \ldots\ldots, C_n$; the centres lie along positive x-axis with abscissae forming an arithmetic sequence of first term unity and common difference 3. The radius of these circles are in geometric sequence with first term unity and common ratio 2. If the tangent lines with slope m_1 and m_2 of C_3 are intersected at the centre of C_5, then compute the value of $5\,|m_1 m_2|$.

3. A sequence is obtained by deleting all perfect squares from set of natural numbers. The remainder when the 2003rd term of new sequence is divided by 2048, is ______.

4. Let $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$ and $10B=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{bmatrix}$

If B is the inverse of the matrix A, then α is ______.

5. $A = (-3, 0)$ and $B = (3, 0)$ are the extremities of the base AB of triangle PAB. If the vertex P varies such that the internal bisector of angle APB of the triangle divides the opposite side AB into two segments AD and BD such that $AD : BD = 2 : 1$, then the maximum value of the length of the altitude of the triangle drawn through the vertex P is ______.

6. Find the natural number c for which $\sum_{r=1}^{n} f(c+r)=16(2^n-1)$ where the function satisfies the relation $f(x + y) = f(x)\cdot f(y)$ for all $x, y \in N$ (natural numbers) and $f(1) = 2$.

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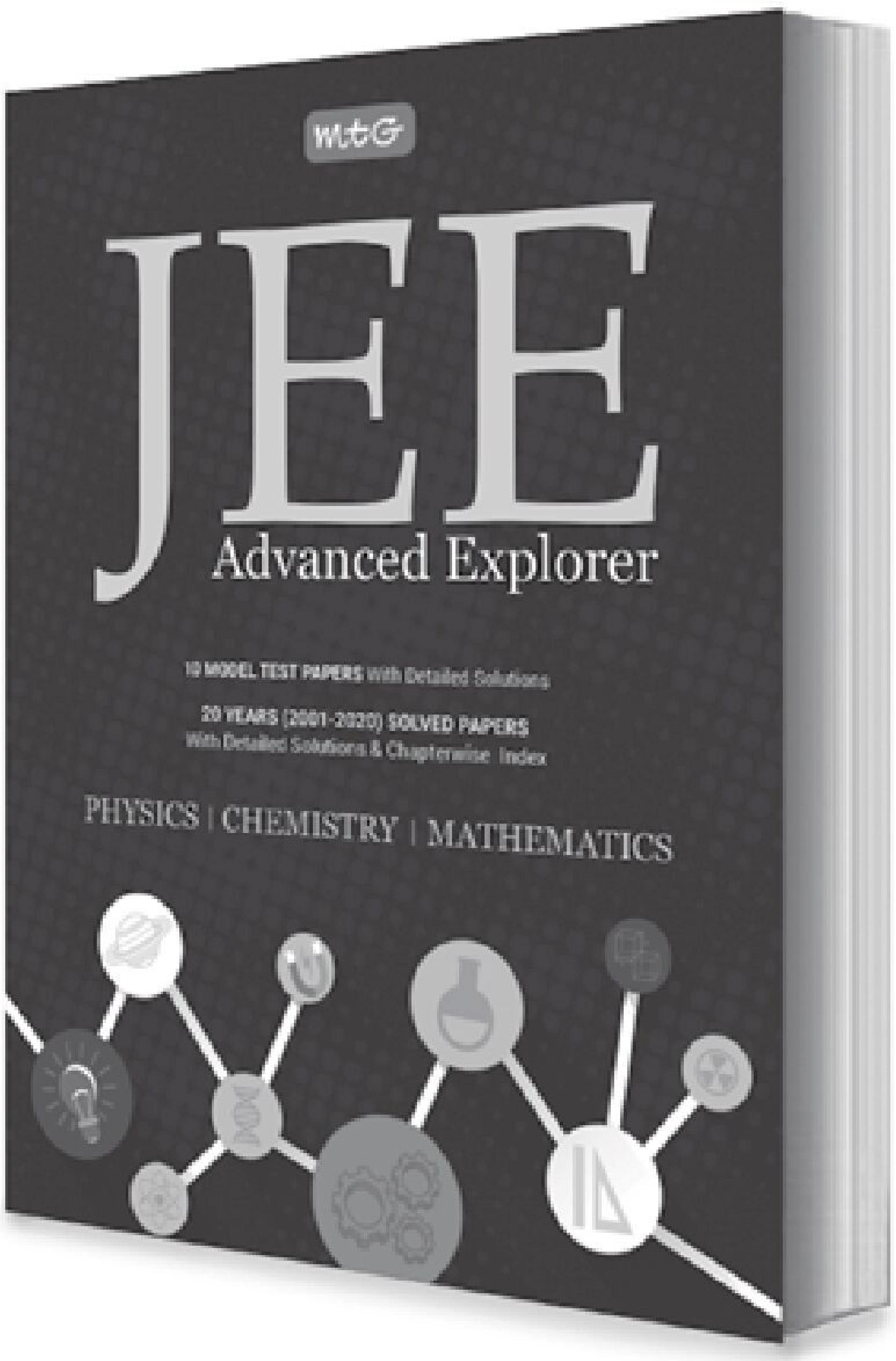
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ONE OR MORE THAN ONE OPTION(S) CORRECT TYPE

7. Let $C_1 = x^2 + y^2 = r_1^2$, $C_2 = x^2 + y^2 = r_2^2$ $(r_1 < r_2)$ be two circles. Let A be the fixed point $(r_1, 0)$ on C_1 and B be a variable point on C_2. Let the line BA meet the circle C_2 again at E. Then,

(a) the maximum value of BE is $2r_2$.

(b) the minimum value of BE is $2\sqrt{r_2^2 - r_1^2}$.

(c) if O is origin, then, the best possible lower bound for $OA^2 + OB^2 + BE^2$ is $5r_2^2 - 3r_1^2$.

(d) if O is origin, then, the best possible upper bound for $OA^2 + OB^2 + BE^2$ is $r_1^2 + 5r_2^2$.

8. $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$, $C(at_3^2, 2at_3)$ be 3 points on the parabola $y^2 = 4ax$. If the orthocentre of ΔABC is focus S of the parabola, then

(a) $t_1t_2 + t_3t_2 + t_3t_1 = -5$

(b) $\frac{1}{t_1t_2}+\frac{1}{t_2t_3}+\frac{1}{t_3t_1}=-1$

(c) If $t_1 = 0$, then $t_2 + t_3 = 0$

(d) $(1 + t_1)(1 + t_2)(1 + t_3) = -4$

9. In a four-dimensional space, where unit vectors along axes are $\hat{i}, \hat{j}, \hat{k}$ and $\hat{l}$ and $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non-zero vectors such that no vector can be expressed as linear combination of others and $(\lambda-1)(\vec{a}_1-\vec{a}_2)$ $+\mu(\vec{a}_2+\vec{a}_3)+\gamma(\vec{a}_3+\vec{a}_4-2\vec{a}_2)+\vec{a}_3+\delta\vec{a}_4=\vec{0}$, then

(a) $\lambda = 1$ (b) $\mu=-\frac{2}{3}$

(c) $\lambda=\frac{2}{3}$ (d) $\delta=\frac{1}{3}$

10. A man wants to divide 101 coins, a rupee each, among his 3 sons with the condition that no one receives more money than the combined total of other two. The number of ways of doing this is

(a) ${}^{103}C_2 - 3\ {}^{52}C_2$ (b) $\frac{{}^{103}C_2}{3}$

(c) 1275 (d) $\frac{{}^{103}C_2}{6}$

11. If $f(x)=\begin{cases} x^2+2, & x<0 \\ 3, & x=0 \\ x+2, & x>0 \end{cases}$, then

(a) $f(x)$ has a maximum at $x = 0$

(b) $f(x)$ is strictly decreasing on the left of 0

(c) $f'(x)$ is strictly increasing on the left of 0

(d) $f'(x)$ is strictly increasing on the right of 0

12. A curve passes through (5, 4) and the slope of tangent at (x, y) is $\frac{1-x}{y-1}$, then

(a) The area of triangle formed by tangent and normal at (5, 4) and the x-axis is 50/3 sq. units

(b) the area of triangle formed by tangent and normal at (5, 4) and the x-axis is 86/3 sq. units

(c) The length of perpendicular from (0, 0) to tangent at (5, 4) is 32/5 units

(d) The length of perpendicular from (0, 0) to tangent at (5, 4) is 16/5 units

NUMERICAL VALUE TYPE

13. If $f(x + y + z) = f(x) + f(y) + f(z)$ with $f(1) = 1$ and $f(2) = 2$ and $x, y, z \in N$, then $\lim\limits_{n\to\infty}\frac{\sum\limits_{r=1}^{n}(4r)f(3r)}{n^3}$ is equal to ________.

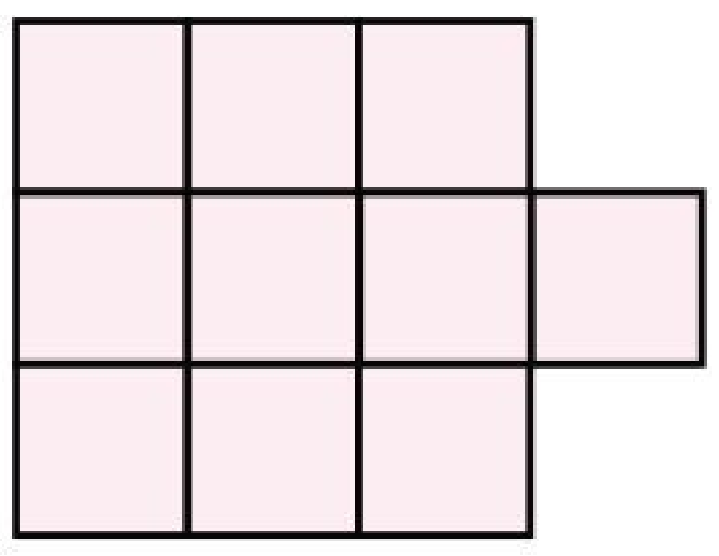
14. Let $\alpha, \beta \in R$. If α, β^2 be the roots of quadratic equation $x^2 - px + 1 = 0$ and α^2, β be the roots of quadratic equation $x^2 - qx + 8 = 0$, then the value of 'r' if $\frac{r}{8}$ be arithmetic mean of p and q, is ________.

15. Let $A=\begin{bmatrix}1 & \frac{3}{2}\\ 1 & 2\end{bmatrix}, B=\begin{bmatrix}4 & -3\\ -2 & 2\end{bmatrix}$

and $C_r=\begin{bmatrix} r\cdot 3^r & 2^r \\ 0 & (r-1)3^r\end{bmatrix}$ be given matrices.

If $\sum\limits_{r=1}^{50} tr((AB)^r C_r)=3+a\cdot 3^b$ where $tr(A)$ denotes trace of matrix A, then find the value of $\frac{1}{20}(a + b)$ [Where a and b are relatively prime].

16. The number of ways in which 7 different letters can be arranged in the given figure such that no row is empty is $59k \cdot 7!$. Then k is ________.



17. A nine digit number is formed from 1, 2, 3, 4, 5 such that the product of all digits is always 1920. The total number of ways is $393({}^9P_r)$, where the value of r is ________.

18. For $r = 0, 1, \ldots, 10$, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansion of

$(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$, then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to $C_p - B_p$, where $p =$ ______.

SOLUTIONS

PAPER-1

1. **(b) :** $z^9(11z+10i) = 11-10iz \Rightarrow z^9 = \dfrac{11-10iz}{11z+10i}$

$$\Rightarrow |z^9| = \frac{|11i+10z|}{|11z+10i|}$$

Now, $|11i+10z|^2 - |11z+10i|^2 = 21(1-|z|^2)$

If $|z|<1$, then $|z^9|>1$ (not possible)

$\Rightarrow$ If $|z|>1 \Rightarrow |z^9|<1$ (not possible)

$\Rightarrow |z| = 1$

2. **(b) :** $\int_1^2 xf'(x)\,dx = xf(x)\Big|_1^2 - \int_1^2 f(x)\,dx$

So,

$$\int_1^2 (f(x)+xf'(x))dx + \int_2^3 f'(x)f''(x)\,dx = xf(x)\Big|_1^2 + \frac{(f'(x))^2}{2}\Bigg|_2^3$$

$$= 2f(2) - 1f(1) + \frac{1}{2}[(f'(3))^2 - (f'(2))^2]$$

$$= 2f(2) - f(1) + \frac{1}{2}[3-1] = 2f(2) - f(1) + 1$$

3. **(a) :** $I_1 = \displaystyle\int_{-100}^{101} \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$

$$\Rightarrow I_1 = \int_{-100}^{101} \frac{dx}{(5+2(1-x)-2(1-x)^2)(1+e^{2-4(1-x)})}$$

$$\Rightarrow 2I_1 = \int_{-100}^{101} \frac{dx}{5+2x-2x^2} = I_2 \Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$$

4. **(b) :** Here, $\tan A + \tan B = \dfrac{c^2}{ab}$ and $\tan A \cdot \tan B = 1$

Now, $\tan A \cdot \tan B = 1 \Rightarrow \tan A = \cot B$

$\Rightarrow A = 90° - B \Rightarrow A + B = 90° \Rightarrow C = 90°$

$\Rightarrow \sin A = \dfrac{a}{c}$, $\sin B = \dfrac{b}{c}$

$$\therefore \sin^2 A + \sin^2 B + \sin^2 C = \frac{a^2}{c^2} + \frac{b^2}{c^2} + 1 = 2$$

5. **(a) :** Total number of outcomes = ${}^{64}C_3$.
Let 'E' be the event of selecting 3 squares which form the letter 'L'.
No. of ways of selecting squares consisting of 4 unit squares = ${}^7C_1 \times {}^7C_1 = 49$
Each square with 4 unit squares forms 4L-shapes consisting of 3 unit squares.

$$\therefore n(E) = 7\times 7\times 4 = 196, \quad \therefore P(E) = \frac{196}{{}^{64}C_3}$$

6. **(a) :** $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

$\Rightarrow c^2 - ab = 0$
and from $ax^2 + 2cx + b = 0$, we have
$D = 4c^2 - 4ab = 4(c^2 - ab) = 0$
So, roots are real and equal.

7. **(a, d) :** For $x>2$, $\displaystyle\int_0^x (5+|1-t|)dt = \int_0^1 (6-t)dt + \int_1^x (4+t)\,dt$

$$= 1 + 4x + \frac{x^2}{2}$$

$$\Rightarrow f(x) = \begin{cases} 1+4x+\dfrac{x^2}{2}, & x>2 \\ 5x+1, & x\le 2 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 4+x, & x>2 \\ 5, & x\le 2 \end{cases}$$

$f(2^+) = f(2^-) = 11 \Rightarrow f(x)$ is continuous at $x = 2$
$f'(2^+) \ne f'(2^-) \Rightarrow f(x)$ is not differentiable at $x = 2$

8. **(a, b) :** Let equation of required plane is
$lx + my + \lambda z = 0$ ...(1)
Angle between (1) and $lx + my = 0$ is α.

$$\Rightarrow \cos\alpha = \frac{l^2+m^2}{\sqrt{l^2+m^2}\sqrt{l^2+m^2+\lambda^2}}$$

$$\Rightarrow \cos^2\alpha = \frac{l^2+m^2}{l^2+m^2+\lambda^2} \Rightarrow \lambda = \pm\sqrt{l^2+m^2}\tan\alpha$$

Hence, equation of plane is

$$lx + my \pm z\sqrt{l^2+m^2}\tan\alpha = 0$$

9. **(a, b) :** $|z_1+z_2| = \left|\dfrac{-b}{a}\right|$; $z_1z_2 = \left|\dfrac{c}{a}\right|$

$$\therefore |z_1+z_2|^2 = 1 \Rightarrow (z_1+z_2)(\bar{z}_1+\bar{z}_2) = 1$$

$$\Rightarrow 2 + \bar{z}_1z_2 + \bar{z}_2z_1 = 1 \Rightarrow \frac{(z_1+z_2)^2}{z_1z_2} = 1$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{c}{a} \Rightarrow b^2 = ac$$

Now, $z_2 = z_1 e^{i\theta}$ then $|z_1 + z_2| = |z_1||1 + e^{i\theta}|$

$$\Rightarrow \quad 2\cos\frac{\theta}{2} = 1 \quad \therefore \quad \theta = \frac{2\pi}{3}$$

$$\therefore \quad PQ = |z_2 - z_1| = \sqrt{3}$$

10. (a, b, c, d) : $\because f_n(\alpha) = \tan(\alpha/2)\,(1 + \sec\alpha)\,(1 + \sec 2\alpha)$ $(1 + \sec 4\alpha)$ ..... $(1 + \sec 2^n\alpha)$

$$= \tan\left(\frac{\alpha}{2}\right)\left(\frac{1+\cos\alpha}{\cos\alpha}\right)(1+\sec 2\alpha)(1+\sec 4\alpha)....(1+\sec 2^n\alpha)$$

$$= \left(\frac{\sin(\alpha/2)}{\cos(\alpha/2)}\right)\left(\frac{2\cos^2(\alpha/2)}{\cos\alpha}\right)$$

$$(1 + \sec 2\alpha)(1+\sec 4\alpha).....(1+\sec 2^n\alpha)$$

$$= \tan\alpha\left(\frac{1+\cos 2\alpha}{\cos 2\alpha}\right)(1+\sec 4\alpha).....(1+\sec 2^n\alpha)$$

$= \tan 2\alpha(1 + \sec 4\alpha)$ ....$(1 + \sec 2^n\alpha)$

$= \tan 4\alpha(1 + \sec 8\alpha)$ .... $(1 + \sec 2^n\alpha)$

$= \tan 8\alpha(1 + \sec 16\alpha)$ ....$(1 + \sec 2^n\alpha) = \tan 2^n\alpha$

Thus $f_n(\alpha) = \tan 2^n\alpha$

$$\therefore \quad f_2\left(\frac{\pi}{16}\right) = \tan\left(\frac{\pi}{4}\right),\ f_3\left(\frac{\pi}{32}\right) = \tan\frac{\pi}{4},$$

$$f_4\left(\frac{\pi}{64}\right) = \tan\frac{\pi}{4},\ f_5\left(\frac{\pi}{128}\right) = \tan\frac{\pi}{4}$$

$$\therefore \quad f_2\left(\frac{\pi}{16}\right) = f_3\left(\frac{\pi}{32}\right) = f_4\left(\frac{\pi}{64}\right) = f_5\left(\frac{\pi}{128}\right) = 1$$

($\because \tan\pi/4 = 1$)

11. (a, c) : $f(x) = \dfrac{\log x}{x}, \quad f'(x) = \dfrac{1-\log x}{x^2} = 0 \Rightarrow x = e$

So, $f(x)$ is many-one function for $x > 1$.

So, $a \in (1, e) \Rightarrow a = 2$

and $\dfrac{\log b}{b} = \dfrac{\log 2}{2} \Rightarrow b = 4$

12. (a, c) : Let ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with $S_1\ (ae, 0)$, $S_2\ (-ae, 0)$ and $P(a\cos\theta, b\sin\theta)$ then $S_1P = a(1 - e\cos\theta)$, $S_2P = a(1 + e\cos\theta)$

So, incentre $(h, k) \equiv \left(ae\cos\theta, \dfrac{be\sin\theta}{1+e}\right)$

On eliminating θ, $\dfrac{h^2}{a^2e^2} + \dfrac{k^2}{b^2e^2}(1+e)^2 = 1$

Thus, locus is an ellipse with eccentricity

$$e_1 = \sqrt{1 - \frac{b^2}{a^2(1+e)^2}} = \sqrt{\frac{2e}{1+e}}$$

13. (85) : Let F_1 and F_2 be the foci and P be the point of tangency. Let F_1' be the image of F_1 in x-axis, then $F_1P + F_2P = F_1'P + F_2P = F_1'F_2 = 2a$

$$= \sqrt{(40)^2 + (75)^2} = 85$$

14. (6) : Let a be the first term of A.P. and d be the common difference.

$$S(3n) = \frac{3n}{2}\{2a + (3n-1)\,d\}, \quad \frac{S(3n)}{S(2n) - S(n)} = 3 = f(n)$$

$$\Rightarrow \quad S = \lim_{n\to\infty}\frac{1}{n}\sum_{r=-n}^{n} f(r) = \lim_{n\to\infty} 3\left(\frac{2n+1}{n}\right) = 6\,.$$

15. (2) : $e^3 = \exp\lim\limits_{x\to 0}\left(\dfrac{x + \dfrac{f(x)}{x}}{x}\right)$

$$\Rightarrow \quad 3 = \lim_{x\to 0} 1 + \frac{f(x)}{x^2} \quad \Rightarrow \quad \lim_{x\to 0}\frac{f(x)}{x^2} = 2$$

16. (6305) : Sum of the coefficients of the terms not containing c is 3^8 and of the terms not containing b and c both is 2^8, so required sum $= 3^8 - 2^8$.

17. (21) : Dividing the given equation by $(20)^{19}$, we get $m = 1 + 2x + 3x^2 + \ldots\ldots + 20x^{19}$,

where $x = \dfrac{21}{20}$.

$$m = \frac{d}{dx}[x + x^2 + \ldots\ldots + x^{20}] = \frac{d}{dx}\left[\frac{x(x^{20}-1)}{x-1}\right]$$

$$= \frac{(x-1)(21x^{20}-1) - x(x^{20}-1)}{(x-1)^2} = \frac{20x^{21} - 21x^{20} + 1}{(x-1)^2}$$

But $20x^{21} - 21x^{20} = 20\left(\dfrac{21}{20}\right)^{21} - 21\left(\dfrac{21}{20}\right)^{20} = 0$

$$\therefore \quad m = \frac{1}{\left(\frac{21}{20} - 1\right)^2} = 400$$

Hence, $\sqrt{m + 41} = \sqrt{441} = 21$

18. (16) : We have

$$0 \le |\vec{\alpha} + \vec{\beta} + \vec{\gamma} + \vec{\delta}|^2 = |\vec{\alpha}|^2 + |\vec{\beta}|^2 + |\vec{\gamma}|^2 + |\vec{\delta}|^2$$
$$+ 2(\vec{\alpha}\cdot\vec{\beta} + \vec{\beta}\cdot\vec{\gamma} + \vec{\gamma}\cdot\vec{\alpha} + \vec{\alpha}\cdot\vec{\delta} + \vec{\gamma}\cdot\vec{\delta} + \vec{\beta}\cdot\vec{\delta})$$

$$= 4 + 2(\vec{\alpha}\cdot\vec{\beta} + \vec{\beta}\cdot\vec{\gamma} + \vec{\gamma}\cdot\vec{\alpha} + \vec{\alpha}\cdot\vec{\delta} + \vec{\gamma}\cdot\vec{\delta} + \vec{\beta}\cdot\vec{\delta})$$

So, $\vec{\alpha}\cdot\vec{\beta} + \vec{\beta}\cdot\vec{\gamma} + \vec{\gamma}\cdot\vec{\alpha} + \vec{\alpha}\cdot\vec{\delta} + \vec{\gamma}\cdot\vec{\delta} + \vec{\beta}\cdot\vec{\delta} \ge -2$

$$\text{Now } |\vec{\alpha}-\vec{\beta}|^2+|\vec{\beta}-\vec{\gamma}|^2+|\vec{\gamma}-\vec{\delta}|^2+|\vec{\delta}-\vec{\alpha}|^2 +|\vec{\gamma}-\vec{\alpha}|^2+|\vec{\beta}-\vec{\delta}|^2$$

$$=3(|\vec{\alpha}|^2+|\vec{\beta}|^2+|\vec{\gamma}|^2+|\vec{\delta}|^2)$$

$$-2(\vec{\alpha}\cdot\vec{\beta}+\vec{\gamma}\cdot\vec{\delta}+\vec{\delta}\cdot\vec{\alpha}+\vec{\alpha}\cdot\vec{\gamma}+\vec{\beta}\cdot\vec{\delta}+\vec{\beta}\cdot\vec{\gamma})\le 3(4)-2(-2)=16$$

PAPER-2

1. (2) : Let the equation be

$ax^2 + bx + c = (\sin\theta)\,x^2 + (\cos\theta)\,x$

$\Rightarrow\ x^2(a - \sin\theta) + x(b - \cos\theta) + c = 0$

Then roots of equation are α, β, γ

Since quadratic has three roots then it must be an identity so.

$a - \sin\theta = 0,\ b - \cos\theta = 0$ and $c = 0$

$\therefore\ a = \sin\theta,\ b = \cos\theta,\ c = 0.$

$$\therefore\ \text{Required value of } \frac{a^2+b^2}{a^2+3ab+5b^2}$$

$$=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta+3\sin\theta\cdot\cos\theta+5\cos^2\theta}$$

$$=\frac{2}{6+4\cos2\theta+3\sin2\theta}$$

For maximum value, denominator should be minimum

$\therefore\ 6 + 4\cos 2\theta + 3\sin 2\theta = 6 - 5 = 1$

$\therefore$ Maximum value of expression is 2.

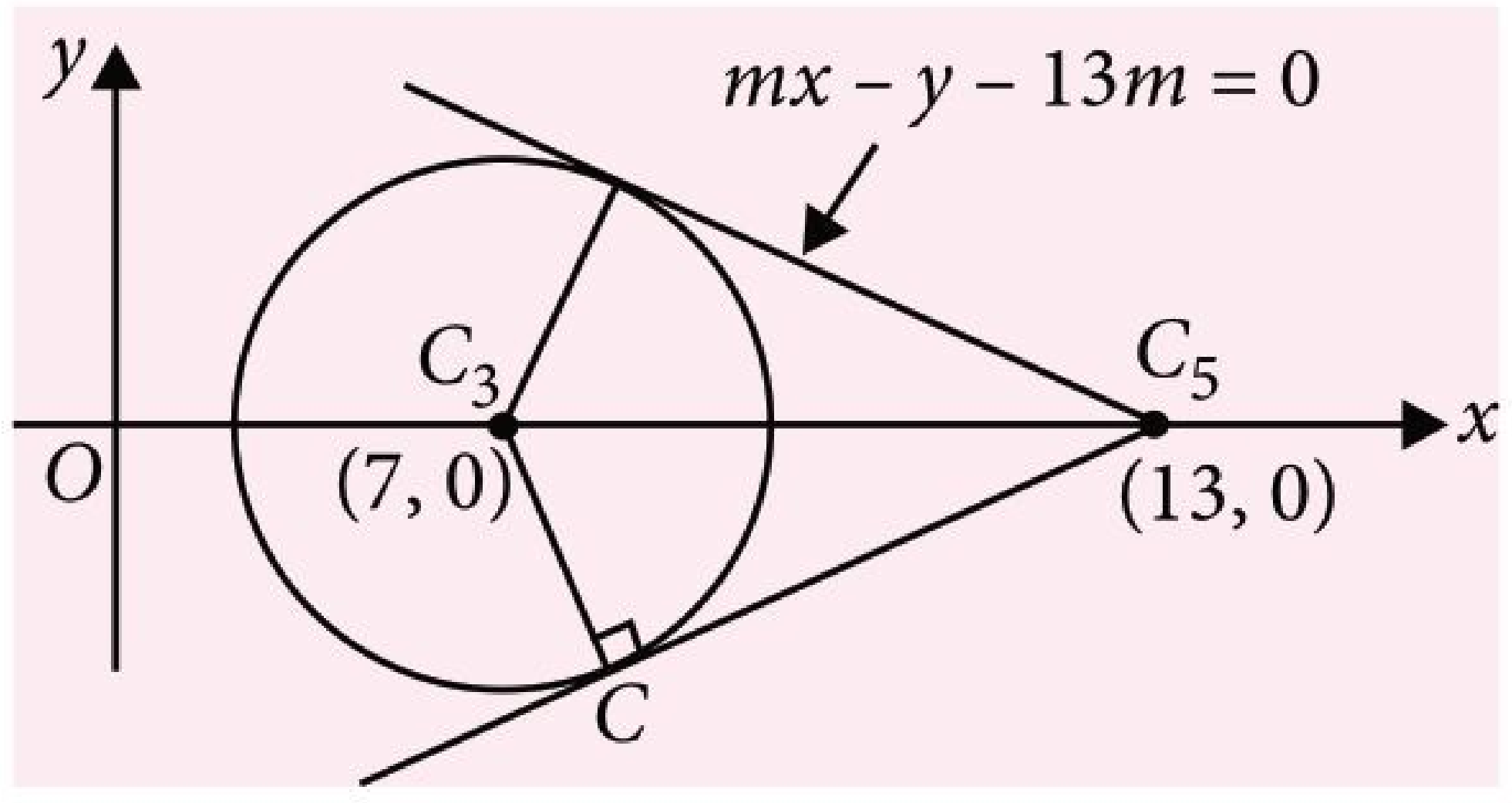
2. (4) : Centre: $C_n = 1 + (n - 1)\,3$; $C_n = 3n - 2$

$C_5 = 13$; $C_3 = 7$; $C_5(13, 0)$; $C_3(7, 0)$

Radius: $R_n = ar^{n-1} = 2^{n-1}$ $\quad\therefore R_3 = 4$

Tangent line :

$y - 0 = m(x - 13) \Rightarrow mx - y - 13m = 0$



$$\Rightarrow \left|\frac{-6m}{\sqrt{m^2+1}}\right|=4 \Rightarrow 9m^2=4m^2+4$$

$$\Rightarrow m^2=\frac{4}{5}\Rightarrow m_1=\frac{2}{\sqrt5} \text{ and } m_2=\frac{-2}{\sqrt5}$$

$$\text{Hence } 5|m_1m_2|=5\times\frac{4}{5}=4$$

3. (0) : Since $\left[\sqrt{2046}\right]=\left[\sqrt{2047}\right]=\left[\sqrt{2048}\right]=\left[\sqrt{2049}\right]=45$

$\therefore$ 2003rd term is 2003 + 45 = 2048

Hence remainder is 0

Remark : There is no perfect square between the numbers m^2 and $(m + 1)^2$.

4. (5) : Since B is the inverse of A $\therefore$ $AB = I$

$$\Rightarrow \begin{bmatrix}1&-1&1\\2&1&-3\\1&1&1\end{bmatrix}\times\frac{1}{10}\begin{bmatrix}4&2&2\\-5&0&\alpha\\1&-2&3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$i.e.,\ \begin{bmatrix}1&-1&1\\2&1&-3\\1&1&1\end{bmatrix}\begin{bmatrix}4&2&2\\-5&0&\alpha\\1&-2&3\end{bmatrix}=\begin{bmatrix}10&0&0\\0&10&0\\0&0&10\end{bmatrix}$$

Comparing (1, 3)th entry on both sides, we get

$2 - \alpha + 3 = 0 \ \Rightarrow\ \alpha = 5.$

5. (4) : The point E dividing $\overrightarrow{AB}$ externally in the ratio 2 : 1 is (9 , 0). Since P lies on the circle described on $\overrightarrow{DE}$ as diameter, coordinates of P are of the form $(5 + 4\cos\theta, 4\sin\theta)$

$\therefore$ Maximum length of the altitude drawn from P to the base $AB = |4\sin\theta|_{\max} = 4.$

6. (3) : From the given equation

$f(x + y) = f(x)\cdot f(y)$, we have $f(x) = e^{kx}$ ...(i)

Where k is a constant.

Putting $x = 1, f(1) = e^k \Rightarrow 2 = e^k$. Hence from (i), $f(x) = 2^x$.

This can also be obtained by putting $y = 1$ so that $f(x + 1) = f(x)\cdot f(1) = 2f(x) \Rightarrow f(x) = 2f(x - 1)$.

Putting successively $x - 1, x - 2, x - 3, \ldots, 2$ for x in the above and multiplying them, we get $f(x) = 2^x$.

$$\text{Now, } \sum_{r=1}^{n} f(c+r) = f(c + 1) + f(c + 2) + \ldots + f(c + n)$$

$$= 2^{c+1} + 2^{c+2} + \ldots + 2^{c+n}$$

$$= 2^c\cdot 2 + 2^c\cdot 2^2 + \ldots + 2^c\cdot 2^n$$

$$= 2^c\cdot 2\{1 + 2 + 2^2 + \ldots \text{ to } n \text{ terms}\}$$

$$= 2^c\cdot 2\cdot\frac{1(2^n-1)}{2-1} = 2^c\cdot 2(2^n - 1)$$

$\Rightarrow\ 16(2^n - 1) = 2^{c+1}(2^n - 1)$

$\Rightarrow\ 2^{c+1} = 16 = 2^4 \Rightarrow c + 1 = 4 \ \therefore\ c = 3.$

7. (a, b, c, d) : $(BE)_{\max}$ = diameter of circle $C_2 = 2r_2$

$$(BE)_{\min}=2\sqrt{r_2^2-r_1^2}$$

$(OA^2 + OB^2 + BE^2)_{\min}$ is

$$r_1^2+r_2^2+4r_2^2-4r_1^2=5r_2^2-3r_1^2$$

$(OA^2 + OB^2 + BE^2)_{\max}$ is

$$r_1^2+r_2^2+4r_2^2=r_1^2+5r_2^2$$

8. (a, b, c) : Slope of $AS = \dfrac{2at_1}{at_1^2 - a} = \dfrac{2t_1}{t_1^2 - 1}$

Slope of $BC = \dfrac{2a(t_3 - t_2)}{a(t_3^2 - t_2^2)} = \dfrac{2}{t_3 + t_2}$

$\therefore \quad \dfrac{2t_1}{t_1^2 - 1} \times \dfrac{2}{t_3 + t_2} = -1$

$\Rightarrow \quad -4t_1 = t_3t_1^2 + t_1^2t_2 - t_3 - t_2$

If $t_1 = 0 \Rightarrow t_3 + t_2 = 0$

So, option (c) is correct.

$$t_1^2(t_2 + t_3) + 4t_1 = t_3 + t_2 \quad \text{...(1)}$$

Similarly $t_2^2(t_1 + t_3) + 4t_2 = t_1 + t_3$ ...(2)

On subtracting (2) from (1), we get

$$t_1^2t_2 + t_1^2t_3 - t_2^2t_1 - t_2^2t_3 + 4(t_1 - t_2) = t_2 - t_1$$

$\Rightarrow \quad t_1t_2(t_1 - t_2) + t_3(t_1^2 - t_2^2) = 5(t_2 - t_1)$

$\Rightarrow \quad t_1t_2 + t_3(t_1 + t_2) = -5$

$\therefore \quad \sum t_1t_2 = -5$

So, option (a) is true

Now, $t_1^2(t_2 + t_3) + 4t_1 = t_3 + t_2$

$\Rightarrow \quad t_1(t_1t_2 + t_3t_1) + 4t_1 = t_2 + t_3$

$\Rightarrow \quad t_1(-5 - t_2t_3) + 4t_1 = t_2 + t_3$

$\Rightarrow \quad -t_1t_2t_3 - t_1 = t_2 + t_3 \Rightarrow -t_1t_2t_3 = t_1 + t_2 + t_3$

$\therefore \quad \dfrac{1}{t_1t_2} + \dfrac{1}{t_2t_3} + \dfrac{1}{t_3t_1} = -1$

So, option (b) is correct.

9. (a, b, d) : $(\lambda - 1)(\vec{a}_1 - \vec{a}_2) + \mu(\vec{a}_2 + \vec{a}_3)$

$$+\gamma(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2) + \vec{a}_3 + \delta\vec{a}_4 = \vec{0}$$

i.e., $(\lambda - 1)\vec{a}_1 + (1 - \lambda + \mu - 2\gamma)\vec{a}_2 + (\mu + \gamma + 1)\vec{a}_3$

$$+(\gamma + \delta)\vec{a}_4 = 0$$

Since $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are linearly independent

$\therefore \quad \lambda - 1 = 0,\ 1 - \lambda + \mu - 2\gamma = 0,\ \mu + \gamma + 1 = 0$

and $\gamma + \delta = 0$

i.e. $\lambda = 1,\ \mu = 2\gamma,\ \mu + \gamma + 1 = 0,\ \gamma + \delta = 0$

i.e. $\lambda = 1,\ \mu = -\dfrac{2}{3},\ \gamma = -\dfrac{1}{3},\ \delta = \dfrac{1}{3}$

10. (a, c) : Let the amount received by the sons be ₹ x, ₹ y and ₹ z respectively, then

$$x \le y + z = 101 - x$$

i.e. $2x \le 101$

$\therefore \quad x \le 50,\ y \le 50,\ z \le 50$

$x + y + z = 101$

The corresponding multinomial is $(1 + x + \ldots + x^{50})^3$

$\therefore \quad$ Coefficient of x^{101} in the expansion of $(1 + x + \ldots + x^{50})^3 = 1 \times {}^{103}C_{101} - 3 \cdot {}^{52}C_{50}$

$$= {}^{103}C_2 - 3 \cdot {}^{52}C_2$$

11. (a, b, c) : $f(x) = \begin{cases} x^2 + 2, & x < 0 \\ 3, & x = 0 \\ x + 2, & x > 0 \end{cases}$

$f(0) = 3, \quad \lim\limits_{x \to 0^-} f(x) = 2, \quad \lim\limits_{x \to 0^+} f(x) = 2$

$\therefore \quad f(x)$ has a maximum at $x = 0$

$f'(x) = 2x,\ x < 0$

$\therefore \quad f'(x) < 0$ for $x < 0$

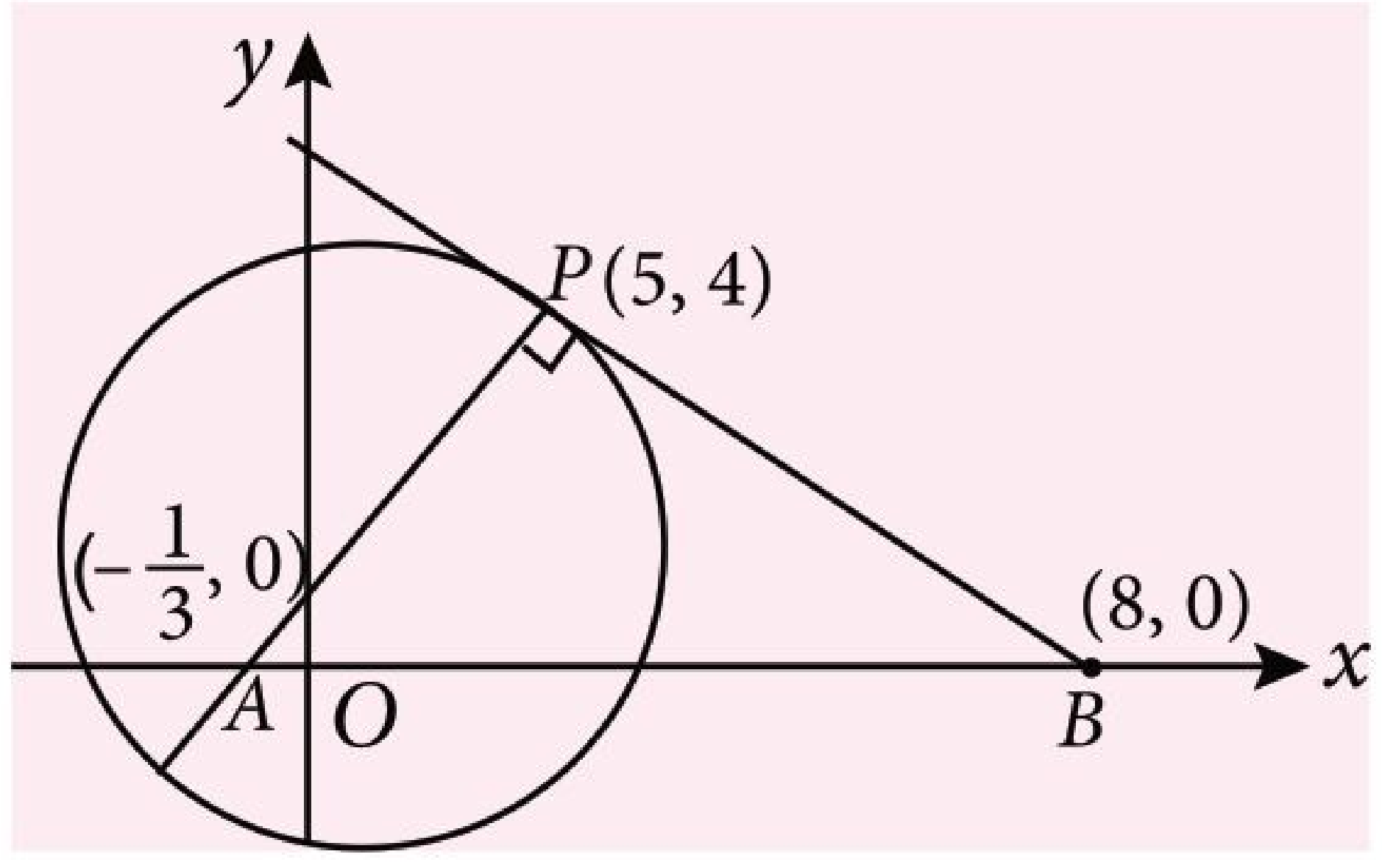
$\therefore \quad f(x)$ is decreasing on the left of 0

$f''(x) = 2,\ x < 0$

$\therefore \quad f''(x) > 0,\ x < 0$

$\therefore \quad f'(x)$ is increasing on the left of 0.

12. (a, c) :



$\dfrac{dy}{dx} = \dfrac{1 - x}{y - 1} \Rightarrow \dfrac{(y - 1)^2}{2} = \dfrac{-(x - 1)^2}{2} + C$

$\because \quad$ It passes through (5, 4)

$\therefore \quad (x - 1)^2 + (y - 1)^2 = 25$

$\Rightarrow \quad x^2 + y^2 - 2x - 2y - 23 = 0$

Equation of tangent at (5, 4) is

$$5x + 4y - (x + 5) - (y + 4) - 23 = 0$$

$\Rightarrow \quad 4x + 3y - 32 = 0$ ...(i)

and equation of normal at (5, 4) is $y - 4 = \dfrac{4 - 1}{5 - 1}(x - 5)$

$\Rightarrow \quad y - 4 = \dfrac{3}{4}(x - 5)$

$\Rightarrow \quad 3x - 4y + 1 = 0$ ... (ii)

$\therefore \quad$ Area of $\Delta ABP = \dfrac{1}{2} \times 4 \times \left(8 + \dfrac{1}{3}\right) = \dfrac{50}{3}$ sq. units

$\therefore \quad$ Length of perpendicular from (0, 0) to (i) is $\left|\dfrac{0 + 0 - 32}{5}\right| = \dfrac{32}{5}$ units

13. (4) : $f(3) = 3f(1) = 3,\ f(4) = f(2 + 1 + 1)$

$= 2 + 1 + 1 = 4$ and so on.

In general, we get $f(r) = r$ for all $r \in N$.

$$\Rightarrow \lim_{n \to \infty} \frac{\sum\limits_{r=1}^{n} (4r) f(3r)}{n^3} = \lim_{n \to \infty} \frac{12n(n + 1)(2n + 1)}{6n^3} = 4$$

14. (83) : For the equation $x^2 - px + 1 = 0$, the product of roots, $\alpha\beta^2 = 1$
and for the equation $x^2 - qx + 8 = 0$,
the product of roots $\alpha^2\beta = 8$
Hence, $(\alpha\beta^2)(\alpha^2\beta) = 8 \Rightarrow \alpha^3\beta^3 = 8 \Rightarrow \alpha\beta = 2$

$\therefore$ From $\alpha\beta^2 = 1$, we have $\beta = \frac{1}{2}$ and from $\alpha^2\beta = 8$, we have $\alpha = 4$

Now, $p = \alpha + \beta^2 = 4 + \frac{1}{4} = \frac{17}{4}$

and $q = \alpha^2 + \beta = 16 + \frac{1}{2} = \frac{33}{2}$

$\because$ $\frac{r}{8}$ is arithmetic mean of p and q

$$\therefore \frac{r}{8} = \frac{p+q}{2} \Rightarrow r = 4(p+q) = 4\left(\frac{17}{4} + \frac{33}{2}\right)$$

$$= 17 + 66 = 83$$

15. (5) : $AB = \begin{bmatrix} 1 & \frac{3}{2} \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\therefore$ $(AB)^1C_1 = C_1$, $(AB)^2C_2 = C_2$ and so on.
Also, $tr(C_r) = r \cdot 3^r + (r-1) \cdot 3^r = (2r-1) \cdot 3^r$

$$\text{Now, } \sum_{r=1}^{50} tr((AB)^r C_r) = tr((AB)^1C_1) + tr((AB)^2C_2) + \ldots\ldots + tr((AB)^{50}C_{50}) = S \text{ (say)}$$

$$\therefore \quad S = tr(C_1) + tr(C_2) + \ldots + tr(C_{50})$$
$$S = 1 \cdot 3^1 + 3 \cdot 3^2 + 5 \cdot 3^3 + \ldots\ldots + 99 \cdot 3^{50}$$
$$3S = 1 \cdot 3^2 + 3 \cdot 3^3 + \ldots\ldots + 97 \cdot 3^{50} + 99 \cdot 3^{51}$$
$$\Rightarrow \quad -2S = 1 \cdot 3 + 2 \cdot 3^2 + 2 \cdot 3^3 + \ldots\ldots + 2 \cdot 3^{50} - 99 \cdot 3^{51}$$
$$= -3 + 2 \cdot 3 + 2 \cdot 3^2 + \ldots\ldots + 2 \cdot 3^{50} - 99 \cdot 3^{51}$$
$$= -3 + 2 \cdot \frac{3 \cdot (3^{50} - 1)}{3-1} - 99 \cdot 3^{51} = -3 + 3^{51} - 3 - 99 \cdot 3^{51}$$
$$= -6 - 98 \cdot 3^{51} \Rightarrow S = 3 + 49 \cdot 3^{51}$$
$$\therefore \quad a + b = 100$$

Hence $\frac{1}{20}(a+b) = 5$

16. (2) : Required number of ways
= total – when one row is empty (either 1st or 3rd)
$= {}^{10}C_7 \cdot 7! - {}^{2}C_1 \cdot 7!$

$$= 7!\left(\frac{10!}{7!3!} - 2\right) = 118 \cdot 7! \Rightarrow k = 2$$

17. (2) : $1920 = 5 \times 3 \times 2^7$
$1920 = 5 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\therefore$ Number of ways $= \frac{9!}{7!} = 9 \times 8$

$1920 = 5 \times 3 \times 4 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1$

$\therefore$ Number of ways $= \frac{9!}{5!} = 9 \times 8 \times 7 \times 6$

$1920 = 3 \times 5 \times 4 \times 4 \times 2 \times 2 \times 2 \times 1 \times 1$

$\therefore$ Number of ways $= \frac{9!}{2!2!3!}$

$1920 = 3 \times 5 \times 4 \times 4 \times 4 \times 2 \times 1 \times 1 \times 1$

$\therefore$ Number of ways $= \frac{9!}{3!3!}$

So, total number of ways $= \frac{9!}{7!} + \frac{9!}{5!} + \frac{9!}{2!2!3!} + \frac{9!}{3!3!}$

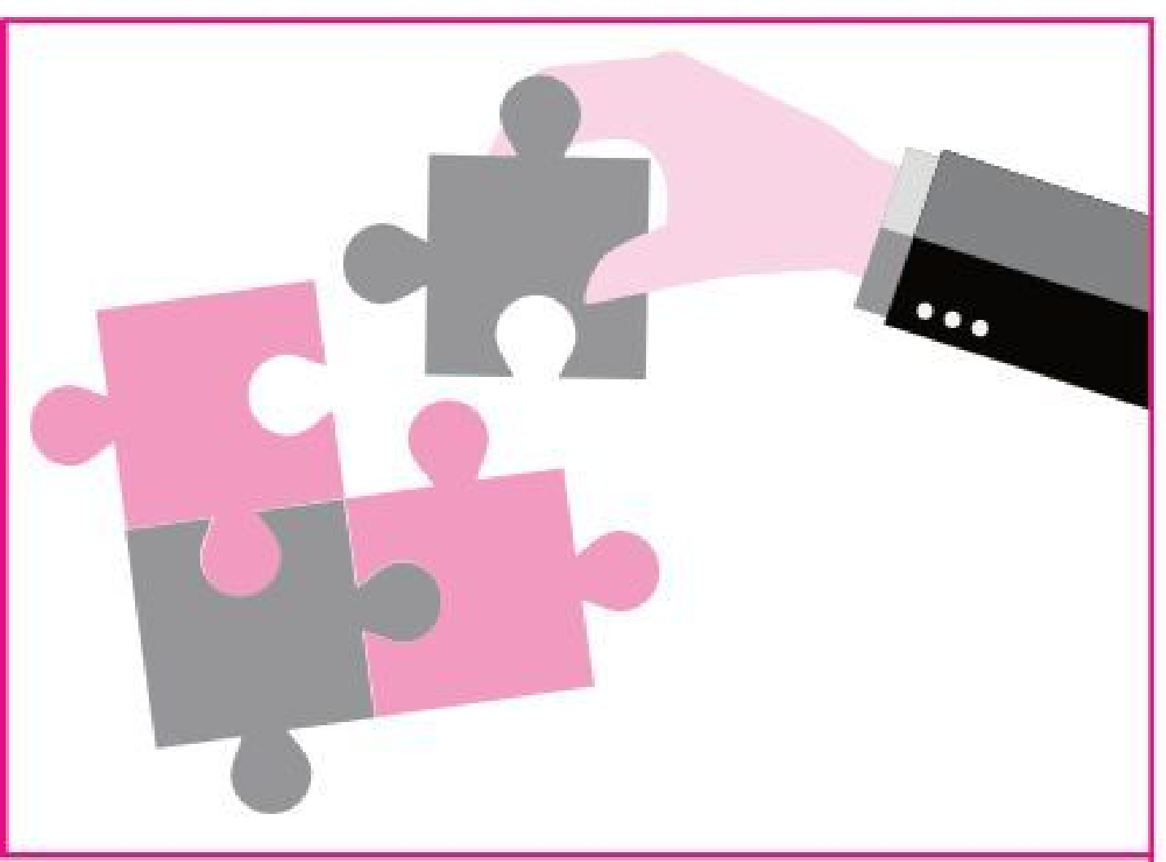
$$= \frac{9!}{7!}[1 + 42 + 210 + 140] = 393 \cdot {}^9P_2$$

18. (10) : $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$

$$= \sum_{r=1}^{10} {}^{10}C_r\,({}^{20}C_{10}\,{}^{20}C_r - {}^{30}C_{10}\,{}^{10}C_r)$$
$$= {}^{20}C_{10}\sum_{r=1}^{10} {}^{10}C_{10-r}\,{}^{20}C_r - {}^{30}C_{10}\sum_{r=1}^{10} {}^{10}C_r\,{}^{10}C_{10-r}$$
$$= {}^{20}C_{10}({}^{30}C_{10} - 1) - {}^{30}C_{10}({}^{20}C_{10} - 1) = {}^{30}C_{10} - {}^{20}C_{10}$$
$$= C_{10} - B_{10} \Rightarrow p = 10$$

◆ ◆

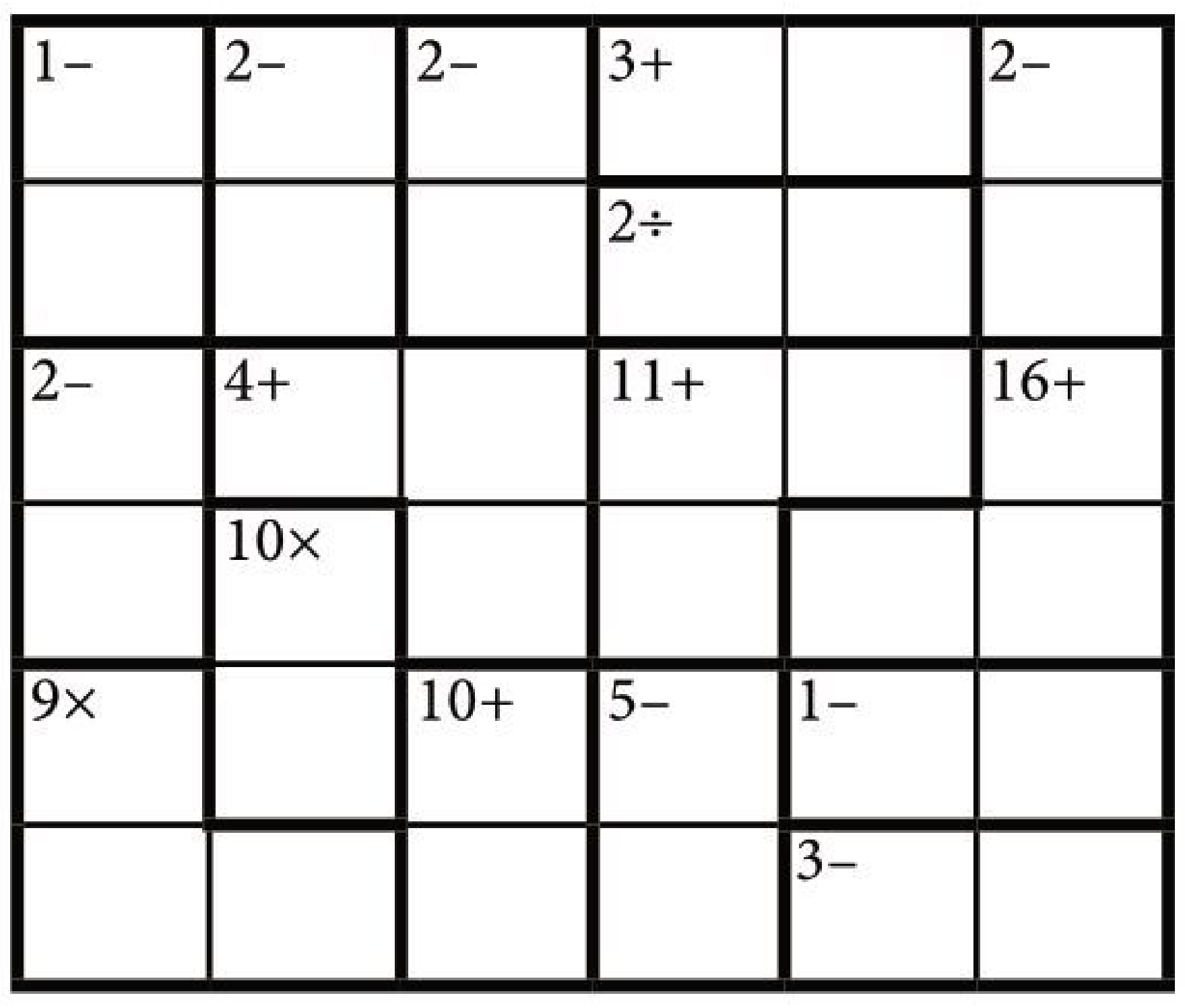
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



1–	2–	2–	3+		2–
			2÷		
2–	4+		11+		16+
	10×				
9×		10+	5–	1–	
				3–	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

PRACTICE PROBLEMS

for JEE 2021

SINGLE OPTION CORRECT TYPE

1. Minimum area of the triangle by any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the coordinate axes is

(a) $\frac{a^2+b^2}{2}$ (b) $\frac{(a+b)^2}{2}$

(c) ab (d) $\frac{(a-b)^2}{2}$

2. A line is drawn through a fixed point $P(\alpha, \beta)$ to cut the circle $x^2 + y^2 = r^2$ at A and B. Then $PA \cdot PB$ is equal to

(a) $(\alpha + \beta)^2 - r^2$ (b) $\alpha^2 + \beta^2 - r^2$

(c) $(\alpha - \beta)^2 + r^2$ (d) None of these

3. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{d}$ be a non-zero vector, which is perpendicular to $\vec{a}+\vec{b}+\vec{c}$. Now, if $\vec{d}=(\sin x)(\vec{a}\times\vec{b})+(\cos y)(\vec{b}\times\vec{c})+2(\vec{c}\times\vec{a})$, then minimum value of $x^2 + y^2$ is equal to

(a) π^2 (b) $\frac{\pi^2}{2}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{5\pi^2}{4}$

4. If $f(x)=\int_0^x (1+t^3)^{-1/2}dt$ and $g(x)$ is the inverse of f, then the value of $\frac{g''(x)}{g^2(x)}$ is

(a) 3/2 (b) 2/3 (c) 1/3 (d) 1/2

5. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{9}$ (c) $-\frac{7}{9}$ (d) $-\frac{3}{5}$

6. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is

(a) 468 (b) 469 (c) 484 (d) 485

7. In ΔABC, $\angle ABC = \angle ACB = 40°$. If P is a point in the interior of the triangle such that $\angle PBC = 20°$ and $\angle PCB = 30°$ then

(a) $BP = BA$ (b) $BP = 2BA$

(c) $BP = \frac{1}{2}BA$ (d) $BP = 3BA$

8. If $f(x) : [1, 10] \to Q$ be a continuous function. If $f(x)$ takes rational value for all x and $f(2) = 5$, then the equation whose roots are $f(3)$ and $f(\sqrt{10})$ is

(a) $x^2 - 10x + 25 = 0$ (b) $x^2 - 3x + 2 = 0$

(c) $x^2 - 6x + 5 = 0$ (d) None of these

9. If a chord joining $P(a \sec\theta, a \tan\theta)$, $Q(a \sec\alpha, a \tan\alpha)$ on the hyperbola $x^2 - y^2 = a^2$ is normal at P, then $\tan\alpha =$

(a) $\tan\theta\,(4\sec^2\theta + 1)$ (b) $\tan\theta\,(4\sec^2\theta - 1)$

(c) $\tan\theta\,(2\sec^2\theta - 1)$ (d) $\tan\theta\,(1 - 2\sec^2\theta)$

10. The lengths of two opposite edges of a tetrahedron are a and b. Their shortest distance is d and the angle between them is θ. Then its volume is

(a) $\frac{1}{2}abd\sin\theta$ (b) $\frac{1}{3}abd\cos\theta$

(c) $\frac{1}{6}abd\sin\theta$ (d) $\frac{1}{6}abd\cos\theta$

ONE OR MORE THAN ONE OPTION(S) CORRECT TYPE

11. Consider the limit, $L=\lim_{x\to 0}(\cos x)^{|\sin x|^{-\alpha}}$ as α ranges from $(0, \infty)$, then

(a) $L = 1$ if $0 < \alpha < 2$ (b) $L = e^{-1/2}$ if $\alpha = 2$
(c) $L = 0$ if $\alpha > 2$ (d) None of these

12. If z, w are complex numbers and $|3z| = 5$, then $\left|\dfrac{75 - 27z\bar{w}}{z - w}\right| =$

(a) 12 (b) 15
(c) 45 (d) none of these

13. Thirteen persons are sitting in a row. Number of ways in which four persons can be selected so that no two of them are consecutive is equal to

(a) number of ways in which all the letters of the word "M A R R I A G E" are permutated if no two vowels are never together.
(b) number of numbers lying between 100 and 1000 using only the digits 1, 2, 3, 4, 5, 6, 7 without repetition.
(c) number of ways in which 4 alike chocolates can be distributed among 10 children so that each child getting at most one chocolate.
(d) number of triangles can be formed by joining 12 points in a plane, of which 5 are collinear.

14. The number of positive integer satisfying the inequality ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$ is

(a) 9 (b) 8
(c) 5 (d) None of these

15. Let A, B be two events such that $P(A \cup B) \ge \dfrac{3}{4}$ and $\dfrac{1}{8} \le P(A \cap B) \le \dfrac{3}{8}$. Then

(a) $P(A) + P(B) \le \dfrac{11}{8}$ (b) $P(A) \cdot P(B) \le \dfrac{3}{8}$

(c) $P(A) + P(B) \ge \dfrac{7}{8}$ (d) $P(A) \cdot P(B) > \dfrac{1}{2}$

16. Let $f(x) = \dfrac{(x-5)^p}{\ln(\cos(x-5))^q}$, $4 < x < 6$; p, q are integers such that $p \ne 0$, $q > 0$. If $\lim\limits_{x \to 5^+} f(x) = -1$, then

(a) $p = 2$ (b) $p = -1$
(c) $q = 1$ (d) $q = 2$

17. Consider the planes $P_1 : 2x + y + z + 4 = 0$
$P_2 : y - z + 4 = 0$ and $P_3 : 3x + 2y + z + 8 = 0$.
Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2 respectively. Then,

(a) At least two of the lines L_1, L_2 and L_3 are non-parallel
(b) At least two of the lines L_1, L_2 and L_3 are parallel
(c) The three planes intersect in a line
(d) The three planes form a triangular prism

18. Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : R \to R$ be defined as

$$g(x) = \begin{cases} 0, & \text{if} \quad x < a \\ \displaystyle\int_a^x f(t)\,dt, & \text{if} \quad a \le x \le b \\ \displaystyle\int_a^b f(t)\,dt, & \text{if} \quad x > b \end{cases}$$

Then

(a) $g(x)$ is continuous but not differentiable at a
(b) $g(x)$ is differentiable on R
(c) $g(x)$ is continuous but not differentiable at b
(d) $g(x)$ is continuous and differentiable at either a or b but not both

19. A bag contains four tickets with numbers 112, 121, 211, 222. One ticket is drawn at random from the bag. Let E_i ($i = 1, 2, 3$) denote the event that i^{th} digit on the drawn ticket is 2. Then

(a) E_1, E_2, E_3 are pairwise independent
(b) E_1, E_2 are independent
(c) E_2 and E_3 are not independent
(d) E_1, E_2, E_3 are mutually independent.

20. Let $f(x) = [b^2 + (a-1)b + 2]x - \int (\sin^2 x + \cos^2 x)\,dx$ be an increasing function of $x \in R$ and $b \in R$, then a can take value (s)

(a) 0 (b) 1 (c) 2 (d) 4

NUMERICAL ANSWER TYPE

21. Let $f(\theta) = \begin{vmatrix} \sin^2\theta & \cos^2\theta & 1 + 4\sin 4\theta \\ \sin^2\theta & 1 + \cos^2\theta & 4\sin 4\theta \\ 1 + \sin^2\theta & \cos^2\theta & 4\sin 4\theta \end{vmatrix}$

and the period of $f(\theta)$ is T, then the value of $\dfrac{4}{\pi} \times T$ is ______.

22. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots\ldots + 20(21)^{19} = m(20)^{19}$, then the value of $\sqrt{m + 41}$ is ______.

23. A, B and C respectively take turns tossing a die. A begins, then B and then C and then again A. The probability that C will be the first one to toss a 6 is m/n, where m and n are in reduced form, then sum of digits of m is ______.

24. For the curve $x^2y^3 = c$ (where c is a constant) the portion of the tangent between the axes is divided in the ratio $a : b$, then $a + b$ equals _____.

25. An open box with a square base is to be made from 400 m^2 of lumber. When its volume is maximum, the ratio of an edge of the base to the height is _____.

26. The number of solutions of the equation
$\left|\sin\frac{\pi}{2}x+\cos\frac{\pi}{2}x\right| = \sqrt{\left|(\ln|x|)^3\right|+1}$ is _____.

27. If the perpendicular distance of line
$(1-i)z+(1+i)\bar{z}+3=0$ from $(3 + 2i)$ is $\frac{\lambda}{2}$, then value of λ is _____.

28. A variable plane at a distance of the one unit from the origin cuts the coordinates axes at A, B and C. If the centroid $D(x, y, z)$ of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then find the value of k _____.

29. The locus of the point of intersection of the tangents at the extremities of a chord of the circle $x^2 + y^2 = a^2$ which touches the circle $x^2 + y^2 = 2ax$ is given by $y^2 = k(a - 2x)a$, then k is _____.

30. The minimum distance of $4x^2 + y^2 + 4x - 4y + 5 = 0$ from the line $-4x + 3y = 3$ is _____.

COMPREHENSION TYPE

Paragraph for Q.No. 31 to 33

If $f : R \to [0, \infty)$ be a function satisfying the property $f(x + y) - f(x - y) = f(x)[f(y) - f(-y)]$ for all real x and y, $f'(0) = \log a$, $f(0) = 1$.

31. $f(x)$ is
(a) e^x (b) $2\ln x$ (c) $4x$ (d) a^x

32. $f'(x)$ is
(a) e^x (b) $a^x \log a$ (c) 4 (d) $5x$

33. The solution of differential equation
$$\frac{dy}{dx}=\frac{(\log_a f(x)+\log_a f(y))^2}{[\log_a f(x)+2][\log_a f(y)-2]}$$ is

(a) $\frac{y-2}{x+2}-\frac{1}{2}\ln\left|1+2\left(\frac{y-2}{x+2}\right)\right|=2\log(x+2)+c$

(b) $\frac{y+2}{x-2}-\ln\left|1+\frac{y+2}{x-2}\right|=c$

(c) $(x+2)(y-2)=\ln\left(\frac{x+2}{y-2}\right)+c$

(d) none of these

Paragraph for Q.No. 34 to 36

In an argand plane z_1, z_2 and z_3 are respectively the vertices of an isosceles triangle ABC with $AC = BC$ and $\angle CAB = \theta$. If z_4 is incentre of triangle, Then

34. The value of $\left(\frac{AB}{IA}\right)^2\left(\frac{AC}{AB}\right)$ is

(a) $\frac{(z_2-z_1)(z_1-z_3)}{(z_4-z_1)^2}$ (b) $\frac{(z_2-z_1)(z_3-z_1)}{(z_4-z_1)}$

(c) $\frac{(z_2-z_1)(z_3-z_1)}{(z_4-z_1)^2}$ (d) none of these

35. The value $(z_4 - z_1)^2(1 + \cos\theta)\sec\theta$ is

(a) $(z_2 - z_1)(z_3 - z_1)$ (b) $\frac{(z_2-z_1)(z_3-z_1)}{(z_4-z_1)}$

(c) $\frac{(z_2-z_1)(z_3-z_1)}{(z_4-z_1)^2}$ (d) $(z_2 - z_1)(z_3 - z_1)^2$

36. The value of $(z_2-z_1)^2\tan\theta\cdot\tan\frac{\theta}{2}$ is

(a) $(z_1 + z_2 - z_3)(z_1 + z_2 - 2z_4)$
(b) $(z_1 + z_2 - z_3)(z_1 + z_2 - z_4)$
(c) $-(z_1 + z_2 - z_3)(z_1 + z_2 - 2z_4)$
(d) none of these

Paragraph for Q.No. 37 and 38

For $x \in \left(0, \frac{\pi}{4}\right)$, let $S_n = \sum_{r=1}^{2n}\sin(\sin^{-1}x^{3r-2})$,

$C_n = \sum_{r=1}^{2n}\cos(\cos^{-1}x^{3r-1})$ and $T_n = \sum_{r=1}^{2n}\tan(\tan^{-1}x^{3r})$,

where $n \in N$ and $n \geq 3$

37. The correct order of S_n, C_n and T_n is given by

(a) $S_n > T_n > C_n$ (b) $S_n < C_n < T_n$

(c) $S_n < T_n < C_n$ (d) $S_n > C_n > T_n$

38. The value of 'x' for which $S_n = C_n + T_n$, is

(a) $\sin\left(\frac{\pi}{5}\right)$ (b) $2\sin\left(\frac{\pi}{5}\right)$

(c) $2\sin\left(\frac{\pi}{10}\right)$ (d) $\sin\left(\frac{\pi}{10}\right)$

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(b)	**5.**	(c)
6.	(c)	**7.**	(b,c,d)	**8.**	(a)	**9.**	(a,b)	**10.**	(a,b,c)
11.	(a,b)	**12.**	(c)	**13.**	(b,c)	**14.**	(c)	**15.**	(b)
16.	(a)	**17.**	(36)	**18.**	(3)	**19.**	(1)	**20.**	(1)

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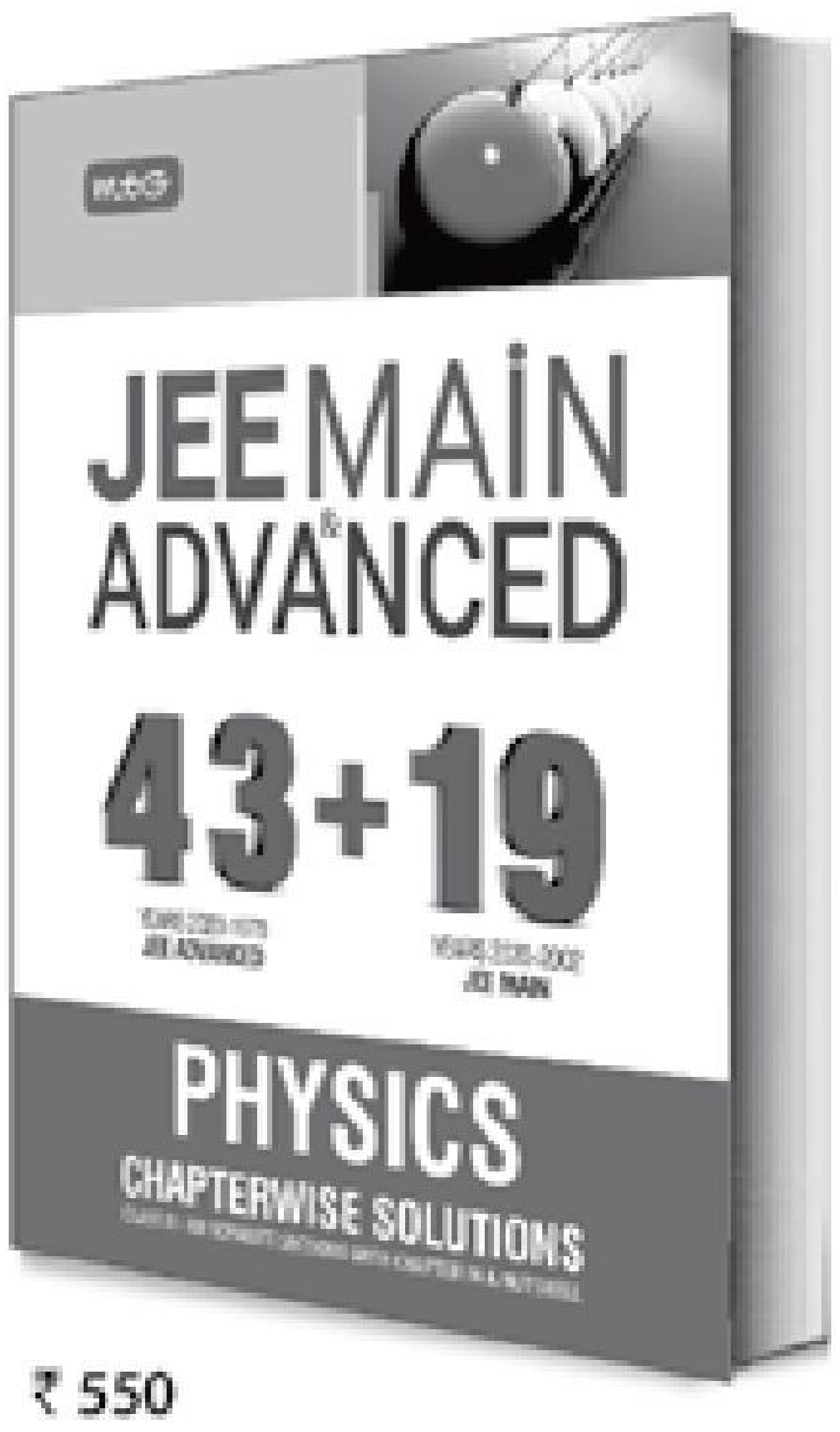
Some of the best lessons are learnt from history!

Introducing Chapterwise Solutions to JEE.
Crack the past to see if you will last.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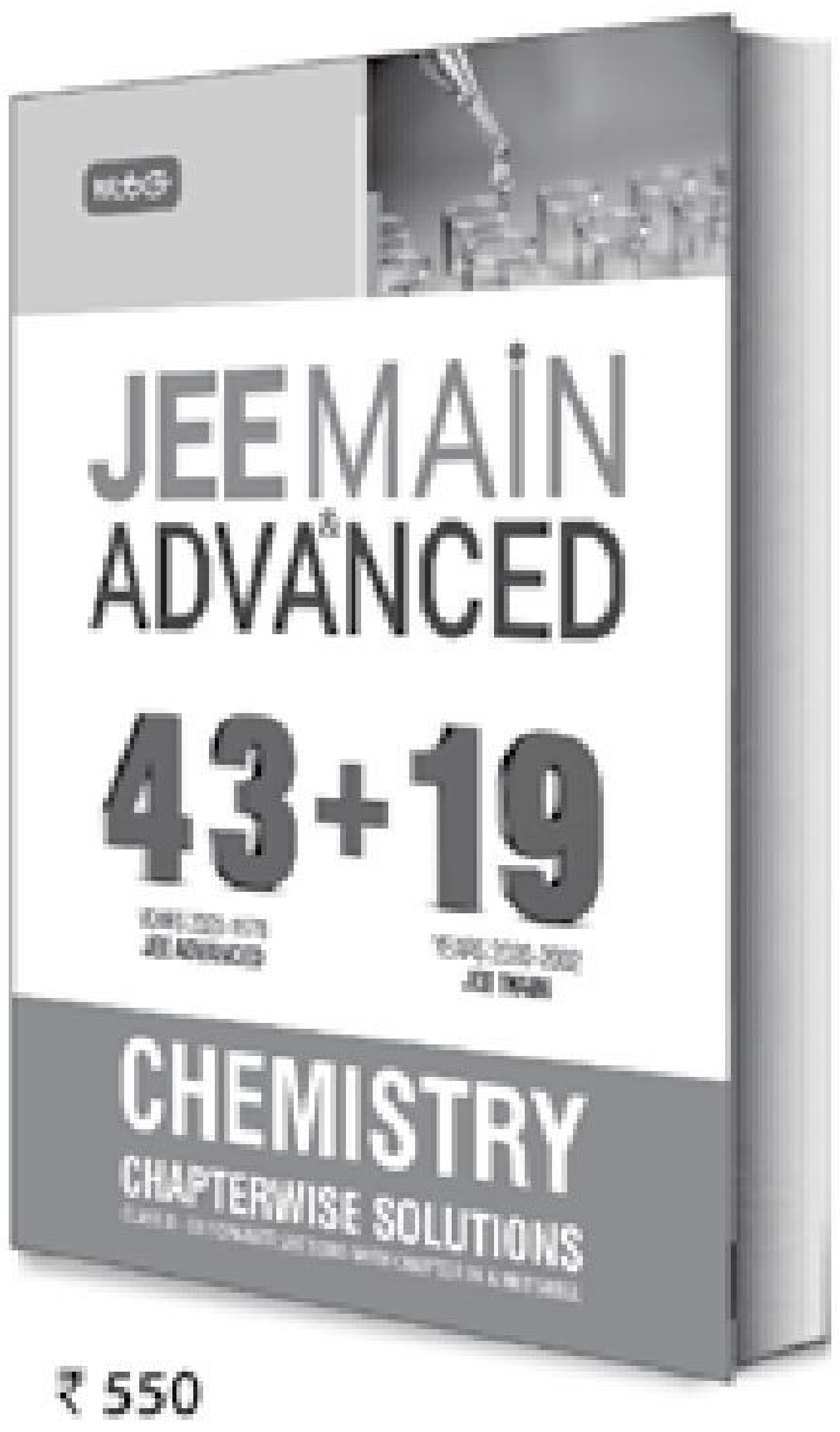
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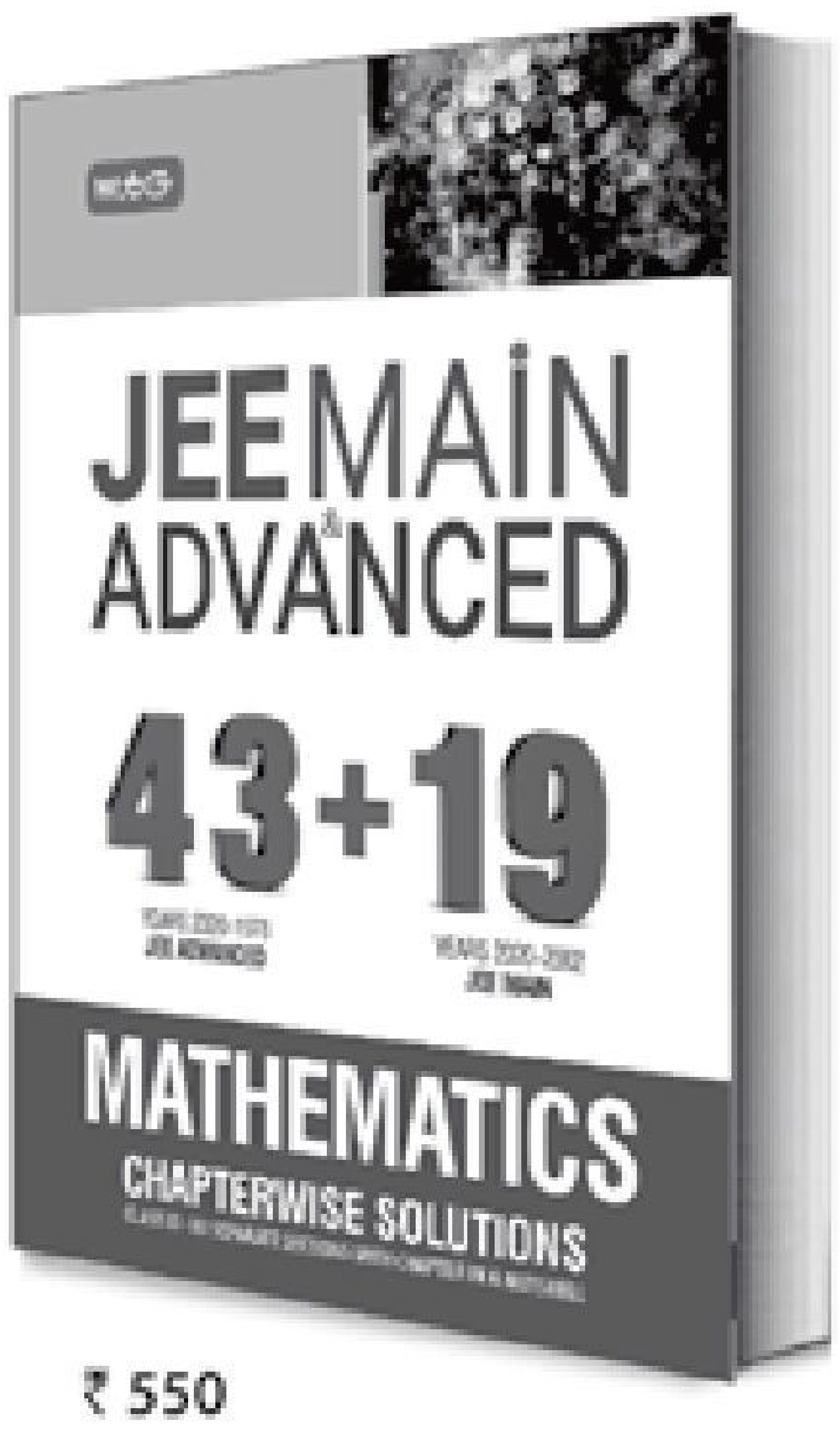
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MATRIX MATCH TYPE

39. Match the following :

Column-I		Column-II	
(A)	If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then the value of $\lvert\vec{a}-\vec{b}\rvert^2+\lvert\vec{b}-\vec{c}\rvert^2+\lvert\vec{c}-\vec{a}\rvert^2$ is at most	(P)	$\sqrt{59}$
(B)	Let $\vec{v}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{w}=\hat{i}+3\hat{k}$. If $\vec{u}$ is a unit vector, then maximum value of the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$ is	(Q)	9
(C)	If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersects then value of k is	(R)	$\sqrt{10}$
		(S)	$\frac{9}{2}$

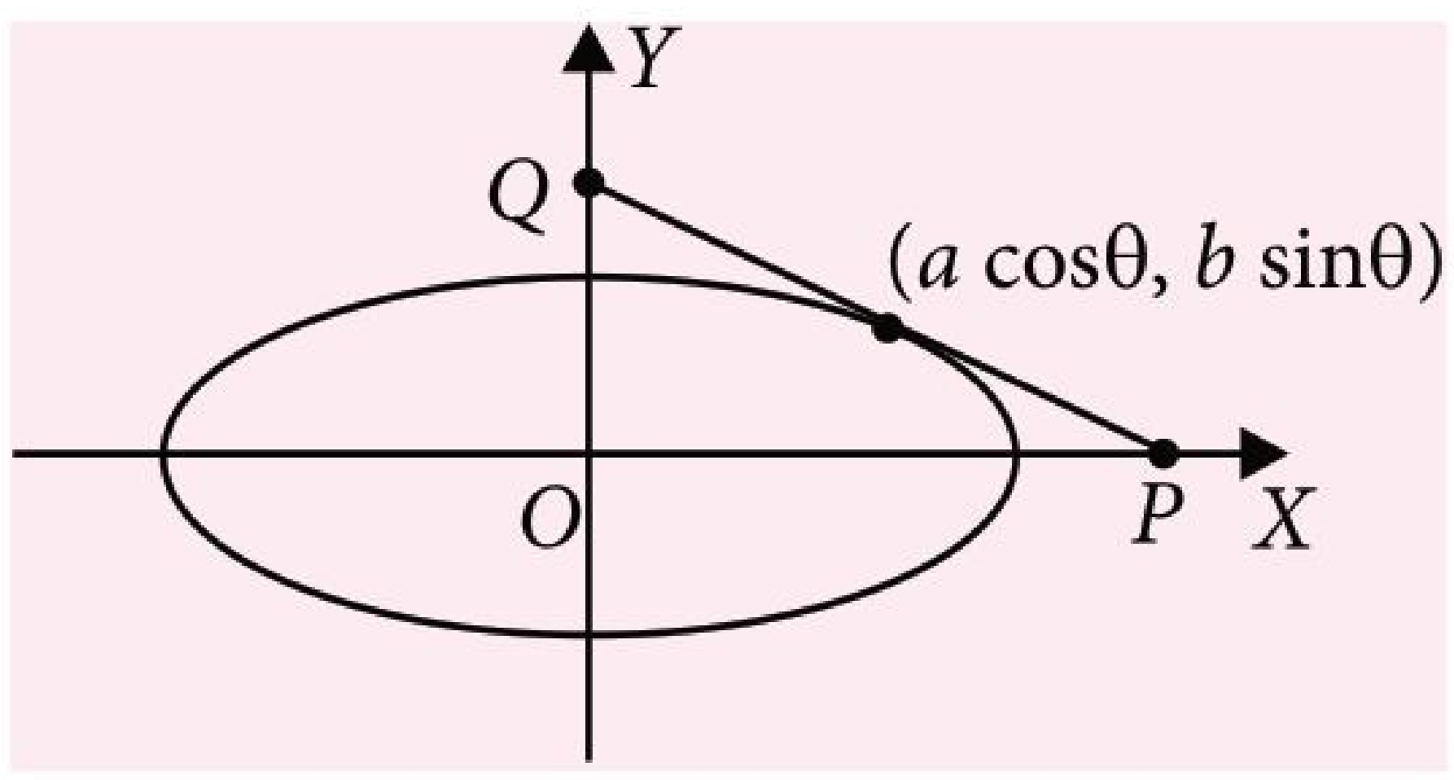
40. Match the following :

Column-I		Column-II	
(A)	$\int\frac{dx}{(a^2-x^2)^2}-\frac{1}{2a^2}\int\frac{dx}{a^2-x^2}=$	(P)	$\frac{x}{2a^2(a^2-x^2)}+c$
(B)	$\int\frac{dx}{x^2\sqrt{x^2-a^2}}$	(Q)	$-\frac{\sqrt{a^2-x^2}}{x}+\cos^{-1}\frac{x}{a}+c$
(C)	$\int\frac{\sqrt{a^2-x^2}}{x^2}dx$	(R)	$\frac{\sqrt{x^2-a^2}}{a^2x}+c$
(D)	$\int\frac{\sqrt{x^2-a^2}}{x}dx$	(S)	$\sqrt{x^2-a^2}-a\sec^{-1}\frac{x}{a}+c$

SOLUTIONS

1. **(c) :** Equation of tangent at (a cosθ, b sinθ) is

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$



$$P=\left(\frac{a}{\cos\theta},0\right)$$

$$Q=\left(0,\frac{b}{\sin\theta}\right)$$

$$\text{Area of } \Delta OPQ=\frac{1}{2}\left|\left(\frac{a}{\cos\theta}\right)\left(\frac{b}{\sin\theta}\right)\right|=\frac{ab}{|\sin 2\theta|}$$

∴ $(\text{Area})_{\min}=ab$.

2. **(b) :** Let the equation of line through the point (α, β) be $\frac{x-\alpha}{\cos\theta}=\frac{y-\beta}{\sin\theta}=k$ (say) ...(i)

where k is the distance of any point (x, y) on the line from the point $P(\alpha, \beta)$. Let this line meets the circle $x^2+y^2=r^2$ at $(\alpha+k\cos\theta, \beta+k\sin\theta)$.

∴ $(\alpha+k\cos\theta)^2+(\beta+k\sin\theta)^2=r^2$

or $k^2+2(\alpha\cos\theta+\beta\sin\theta)k+(\alpha^2+\beta^2-r^2)=0$,

which is a quadratic in k. If k_1 and k_2 are its roots and the line (i) meets circle at A and B, then $PA=k_1$ and $PB=k_2$.

∴ $PA\cdot PB=k_1k_2=$ Products of roots $=\alpha^2+\beta^2-r^2$.

3. **(d) :** Since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar $\Rightarrow [\vec{a}, \vec{b}, \vec{c}]\neq 0$

Also, $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ are non-coplanar.

Given $\vec{d}=(\sin x)(\vec{a}\times\vec{b})+(\cos y)(\vec{b}\times\vec{c})+2(\vec{c}\times\vec{a})$.

Taking dot product with $\vec{a}+\vec{b}+\vec{c}$, we get

$0=\sin x[\vec{a}\ \vec{b}\ \vec{c}]+\cos y[\vec{a}\ \vec{b}\ \vec{c}]+2[\vec{a}\ \vec{b}\ \vec{c}]$

$\Rightarrow \sin x+\cos y+2=0 \Rightarrow \sin x+\cos y=-2$

$\Rightarrow x=(4n-1)\frac{\pi}{2};\ y=(2n+1)\pi, n\in Z$

For least value of x^2+y^2, $x=\frac{-\pi}{2}, y=\pi$

and least value is $\frac{5\pi^2}{4}$.

4. **(a) :** $f(x)=\int_0^x(1+t^3)^{-1/2}dt$

i.e. $f[g(x)]=\int_0^{g(x)}(1+t^3)^{-1/2}dt$

$\Rightarrow x=\int_0^{g(x)}(1+t^3)^{-1/2}dt$

[∵ g is inverse of $f \Rightarrow f[g(x)]=x$]

Differentiating with respect to x, we have

$1=(1+g^3(x))^{-1/2}.g'(x)$

$\Rightarrow (g'(x))^2=1+g^3(x)$

Differentiating again with respect to x, we have

$$2g'(x)g''(x)=3(g(x))^2g'(x) \Rightarrow \frac{g''(x)}{(g(x))^2}=\frac{3}{2}$$

5. **(c) :** $5(\tan^2x-\cos^2x)=2\cos 2x+9$

Let $u=\tan^2x$, we have $5\left(u-\frac{1}{1+u}\right)=2\left(\frac{1-u}{1+u}\right)+9$

$\Rightarrow 5(u^2+u-1)=2-2u+9+9u$

∴ $5u^2-2u-16=0 \Rightarrow (5u+8)(u-2)=0$

But u is positive

$\therefore\ u = 2$

Now, $\tan^2 x = 2 \Rightarrow \cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x} = \frac{1-2}{1+2} = \frac{-1}{3}$

$\Rightarrow \cos 4x = 2\cos^2 2x - 1 = 2\left(\frac{1}{9}\right) - 1 = \frac{-7}{9}$

6. (d) : We can do casework on number of ladies and men to be invited.

X, Y can satisfy the condition in 4 ways

(i) X invites 3 ladies and Y invites 3 men.

(ii) X invites 2 ladies, 1 man and Y invites 1 lady 2 men.

(iii) X invites 1 lady, 2 men and Y invites 2 ladies, 1 man.

(iv) X invites 3 men and Y invites 3 ladies.

The number of ways

$= {}^4C_3 \cdot {}^4C_3 + {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 + {}^3C_3 \cdot {}^3C_3$

$= 16 + 324 + 144 + 1 = 485.$

7. (a) : Let us assume that $AB = AC = 1$ unit, then $BC = 2\cos 40°$ and $\angle BPC = 130°$

Applying sine rule in ΔBPC, we have

$$\frac{BP}{\sin(30°)} = \frac{BC}{\sin(130°)}$$

$$\Rightarrow BP = \frac{BC \cdot \sin 30°}{\cos 40°} = \frac{2\cos 40° \cdot \sin 30°}{\cos 40°} = 1$$

$\Rightarrow BP = AB$

8. (a) : Since $f(x)$ is continuous in $[1, 10]$.

$f(x)$ will attain all values between $f(1)$ and $f(10)$. If $f(1) \neq f(10)$, then $f(x)$ will attain innumerable irrational values between $f(1)$ and $f(10)$.

But given that $f(x)$ attains rational values only, then we must have $f(1) = f(10)$, infact $f(x)$ = constant for $x \in [1, 10]$.

Since $f(2) = 5$, $f(x) = f(2) = 5$, $\forall\, x$.

Hence the equation whose roots are $f(3)$ and $f(\sqrt{10})$ is $x^2 - (5+5)x + 25 = 0$.

9. (b) : Slope of chord joining P and Q = slope of normal at P

$$\frac{\tan\alpha - \tan\theta}{\sec\alpha - \sec\theta} = -\frac{\tan\theta}{\sec\theta} \Rightarrow \tan\alpha - \tan\theta = -k\tan\theta$$

and $\sec\alpha - \sec\theta = k\sec\theta$

$\therefore (1-k)\tan\theta = \tan\alpha$ and $(1+k)\sec\theta = \sec\alpha$

$\Rightarrow [(1+k)\sec\theta]^2 - [(1-k)\tan\theta]^2 = \sec^2\alpha - \tan^2\alpha$

$\Rightarrow k = -2(\sec^2\theta + \tan^2\theta) = -4\sec^2\theta + 2$

$\tan\alpha = \tan\theta(1 + 4\sec^2\theta - 2) = \tan\theta(4\sec^2\theta - 1)$

10. (c) : Consider $OABC$, $\overrightarrow{OA} = \vec{\alpha}$, $\overrightarrow{OB} = \vec{\beta}$, $\overrightarrow{OC} = \vec{\gamma}$

And OA, BC as a pair of opposite edges.

$$|\overrightarrow{OA}| = a, |\overrightarrow{BC}| = b$$

Equation of OA is $\vec{r} = \vec{O} + t\vec{\alpha}$

Equation of BC is $\vec{r} = \vec{\beta} + s(\vec{\beta} - \vec{\gamma})$

$$d = \frac{-[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]}{|\vec{\alpha}||\vec{\beta} - \vec{\gamma}|\sin\theta} \Rightarrow V = \frac{abd\sin\theta}{6}$$

11. (a, b, c) : The given limit is

$$e^{\lim\limits_{x\to 0}\left(\frac{1}{|\sin x|^{\alpha}}\cdot\log(\cos x)\right)}$$

$$= e^{\lim\limits_{x\to 0}\left(\frac{x^2}{|\sin x|^{\alpha}}\cdot\frac{\cos x - 1}{x^2}\cdot\frac{\log(\cos x)}{\cos x - 1}\right)}$$

$= 1$ for $0 < \alpha < 2$

$= e^{-1/2}$ for $\alpha = 2$

$= 0$ for $\alpha > 2$

12. (c) : $|3z| = 5 \Rightarrow |3z|^2 = 25 \Rightarrow 9z\bar{z} = 25$

Now, $\left|\frac{75 - 27z\bar{w}}{z - w}\right| = \left|\frac{27z\bar{z} - 27z\bar{w}}{z - w}\right|$

$$= 27\left|\frac{z(\bar{z} - \bar{w})}{z - w}\right| = 27|z|\left|\frac{\overline{z - w}}{z - w}\right| = 9|3z| = 45$$

13. (b, c, d) : $x_1 + x_2 + x_3 + x_4 + x_5 = 9$, $x_1, x_5 \geq 0$

$x_2, x_3, x_4 \geq 1$, number of solutions are 210.

(a) $5 \times 12 \times 12 = 720$ (b) ${}^7P_3 = 210$

(c) ${}^{10}C_4 = 210$ (d) ${}^{12}C_3 - {}^5C_3 = 210$

14. (b) : ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \leq 100$

$$\Rightarrow {}^{n+1}C_3 - {}^{n+1}C_2 \leq 100 \Rightarrow \frac{1}{6}(n+1)n(n-1) - \frac{n(n+1)}{2} \leq 100$$

$$\Rightarrow \frac{1}{6}n(n+1)\{n-1-3\} \leq 100 \Rightarrow n(n+1)(n-4) \leq 600$$

It is true for $n = 2, 3, 4, 5, 6, 7, 8, 9$

15. (a, c) : $\frac{3}{4} \leq P(A \cup B) \leq 1$

$$\Rightarrow \frac{3}{4} \leq P(A) + P(B) - P(A \cap B) \leq 1$$

$\therefore\ P(A) + P(B) \geq \frac{3}{4} + P(A \cap B) \geq \frac{3}{4} + \frac{1}{8}$ and

$$P(A) + P(B) \leq 1 + P(A \cap B) \leq 1 + \frac{3}{8}$$

16. (a, d) : $\lim\limits_{x\to 5^+} \frac{(x-5)^p}{\ln\cos^q(x-5)} = -1$

$$\Rightarrow \lim_{h\to 0}\frac{h^{p}}{q\ln\cos h}=-1 \Rightarrow \lim_{h\to 0}\frac{ph^{p-1}\cos h}{-q\sin h}=-1$$

$\Rightarrow p-2=0 \Rightarrow p=2$

$\therefore \frac{2}{q}=1 \Rightarrow q=2$

17. (a, c) : Observe that the lines L_1, L_2 and L_3 are parallel to the vector $(1, -1, -1)$.
Also $\Delta = 0 = \Delta_1$ and $b_1c_2 - b_2c_1 \neq 0$
$\because$ Coefficients are not proportional, the three planes intersect in a line

18. (a, c) : $g(a^-) = 0$,

Also, $g(a^+)=\lim\limits_{h\to 0}\int_a^{a+h} f(t)dt=0$

Thus g is continuous at $x = a$. Similarly, g is continuous at $x = b$.

$$g'(a^-)=\lim_{h\to 0}\frac{g(a-h)-g(a)}{-h}=\lim_{h\to 0}\frac{0-\int_a^a f(t)dt}{-h}=\lim_{h\to 0}\frac{0}{-h}=0$$

$$g'(a^+)=\lim_{h\to 0}\frac{g(a+h)-g(a)}{h}=\lim_{h\to 0}\frac{\int_a^{a+h} f(t)dt-0}{h}$$

$$=\lim_{h\to 0}\frac{f(a+h)}{1}=f(a)\geq 1$$

So, $g'(a^-) \neq g'(a^+)$
Hence g is not differentiable at $x = a$.
Similarly, g is not differentiable at $x = b$.

19. (a, b): $P(E_1) = 2/4 = 1/2$, $P(E_2) = 2/4 = 1/2$,
$P(E_3) = 1/2$
$P(E_1E_2) = 1/4$, $P(E_1E_3) = 1/4$
$P(E_2E_3) = 1/4$, $P(E_1E_2E_3) = 1/4$
$\therefore$ $P(E_1E_2) = P(E_1)\cdot P(E_2)$; $P(E_2E_3) = P(E_2)\cdot P(E_3)$;
$P(E_1E_3) = P(E_1)\cdot P(E_3)$
$\therefore$ E_1, E_2, E_3 are pairwise independent
Finally, $P(E_1E_2E_3) = 1/4 \neq P(E_1)\cdot P(E_2)\cdot P(E_3)$
So, E_1, E_2, E_3 are not mutually independent.

20. (a, b, c) : $f'(x) = b^2 + (a-1)b + 2 - \sin^2 x - \cos^2 x$
$\Rightarrow$ $b^2 + (a-1)b + 2 - 1$ for minimum value
$\Rightarrow$ $(a-1)^2 - 4 < 0 \Rightarrow a \in (-1, 3)$.

21. (2) : $f(\theta)=\begin{vmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \end{vmatrix}$

$(R_1 \to R_1 - R_3;\ R_2 \to R_2 - R_3)$

$$=\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 1+\sin^2\theta+4\sin 4\theta & \cos^2\theta & 4\sin 4\theta \end{vmatrix} (C_1 \to C_1 + C_3)$$

$= -(\cos^2\theta + 1 + \sin^2\theta + 4\sin 4\theta) = -2(1 + 2\sin 4\theta)$

which is periodic function with period $\frac{2\pi}{4}=\frac{\pi}{2}$

Hence the value of $\frac{4}{\pi}\times T=\frac{4}{\pi}\cdot\frac{\pi}{2}=2$

22. (21) : Dividing the given equation by $(20)^{19}$, we get
$m = 1 + 2x + 3x^2 + \ldots\ldots + 20x^{19}$, where $x = \frac{21}{20}$.

$$m=\frac{d}{dx}[x+x^2+\ldots\ldots+x^{20}]=\frac{d}{dx}\left[\frac{x(x^{20}-1)}{x-1}\right]$$

$$=\frac{(x-1)(21x^{20}-1)-x(x^{20}-1)}{(x-1)^2}$$

$$=\frac{20x^{21}-21x^{20}+1}{(x-1)^2}$$

But $20x^{21} - 21x^{20} = 20\left(\frac{21}{20}\right)^{21}-21\left(\frac{21}{20}\right)^{20}=0$

$$\therefore\ m=\frac{1}{\left(\frac{21}{20}-1\right)^2}=400$$

Hence, $\sqrt{m+41}=\sqrt{441}=21$

23. (7) : Let P be the probability that C wins then,

$$P=\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^5\times\frac{1}{6}+\left(\frac{5}{6}\right)^8\times\frac{1}{6}+\ldots.$$

$$P=\left(\frac{5}{6}\right)^2\times\frac{1}{6}\left[1+\left(\frac{5}{6}\right)^3+\left(\frac{5}{6}\right)^6+\ \ldots\right]$$

$$P=\left(\frac{5}{6}\right)^2\times\frac{1}{6}\left[\frac{1}{1-\left(\frac{5}{6}\right)^3}\right]=\left(\frac{5}{6}\right)^2\times\frac{1}{6}\times\frac{216}{91}=\frac{25}{91}$$

$\therefore$ $m = 2 + 5 = 7$.

24. (5) : Given curve is $x^2y^3 = c$
Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{-2y}{3x}$$

Equation of tangent at general point (x, y) is

$$Y-y=-\frac{2}{3}\frac{y}{x}(X-x)$$

x-intercept $= \frac{5}{2}x$, y-intercept $= \frac{5}{3}y$.

$$\Rightarrow \quad A \equiv \left(\frac{5}{2}x,\ 0\right) \text{ and } B \equiv \left(0,\ \frac{5}{3}y\right)$$

Let $AP : PB = k : 1$

$$\Rightarrow \quad P \equiv \left(\frac{5x}{2(k+1)},\ \frac{5ky}{3(k+1)}\right)$$

$$\Rightarrow \quad \frac{5x}{2(k+1)} = x \text{ and } \frac{5ky}{3(k+1)} = y$$

(Comparing with $P \equiv (x, y)$)

$$\Rightarrow \quad k = \frac{3}{2} \text{ from both equations.}$$

Thus P divides AB in ratio 3 : 2.

25. (2) : Let x be the length of an edge of the base and y be the height.

Given, $x^2 + 4xy = 400$ ...(i)

$$\text{Volume, } V = x^2 y \ \Rightarrow\ V = \frac{x}{4}(400 - x^2) \ \Rightarrow\ \frac{dV}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{4}(400 - x^2) + \frac{x}{4}(-2x) = 0$$

$$\Rightarrow \quad \frac{-x^2}{4} + 100 - \frac{x^2}{2} = 0$$

$$\Rightarrow \quad x = \frac{20}{\sqrt{3}},\ y = \frac{10}{\sqrt{3}} \quad \therefore \quad \frac{x}{y} = 2$$

26. (6) : $\sin\pi x + 1 = |(\ln|x|)^3| + 1 \Rightarrow \sin\pi x = |(\ln|x|)^3|$

$\therefore$ Number of solutions = 6

27. (13) : We have, $(1-i)z + (1+i)\bar{z} + 3 = 0$

Perpendicular distance of the given line from $(3 + 2i)$

$$= \left|\frac{(1-i)(3+2i) + (1+i)(3-2i) + 3}{\sqrt{(2+2)}}\right|$$

$$= \left|\frac{3 + 2i - 3i + 2 + 3 - 2i + 2 + 3i + 3}{2}\right| = \frac{13}{2} = \frac{\lambda}{2} \text{ (Given)}$$

$\therefore \ \lambda = 13$

28. (9) : Let the equation of variable plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, which meets the axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$.

Centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ and it satisfied the relation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k \ \Rightarrow\ \frac{9}{a^2} + \frac{9}{b^2} + \frac{9}{c^2} = k$

$$\Rightarrow \quad \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{k}{9} \qquad \text{...(i)}$$

Also given that the distance of plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ from(0, 0, 0) is 1 unit.

$$\Rightarrow \quad \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = 1 \ \Rightarrow\ \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 1 \qquad \text{...(ii)}$$

From (i) and (ii) we get $k = 9$

29. (1) : $T \equiv hx + ky - a^2 = 0$

$$\Rightarrow \quad a = \frac{ah + 0 - a^2}{\sqrt{h^2 + k^2}}$$

$\Rightarrow$ $h^2 + k^2 = (h - a)^2 \Rightarrow k^2 = a(a - 2h)$

$\therefore$ The locus is $y^2 = a(a - 2x)$.

$\therefore$ $k = 1$

30. (1) : The given curve represents the point $\left(-\frac{1}{2}, 2\right)$

$\therefore$ Minimum distance = 1.

(31-33) : $f(x + y) - f(x - y) = f(x)[f(y) - f(-y)]$

$$f'(0) = \lim_{h\to 0}\frac{f(h) - f(0)}{h} = \lim_{h\to 0}\frac{f(h) - 1}{h} = \log a$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x-h)}{2h} = \lim_{h\to 0}\frac{f(x)\left[f(h) - f(-h)\right]}{2h}$$

$$= \lim_{h\to 0}\frac{f(x)}{2}\left[\frac{f(h) - 1}{h} + \frac{f(-h) - 1}{-h}\right] = \frac{f(x)}{2}2\log a$$

$$\Rightarrow \quad \frac{f'(x)}{f(x)} = \log a \ \Rightarrow\ \log f(x) = \log a^x + c$$

$f(0) = 1 \Rightarrow c = 0 \Rightarrow f(x) = a^x$

Hence $f'(x) = f(x)\log a = a^x \log a$

$$\text{Now, } \frac{dy}{dx} = \frac{(x + y)^2}{(x + 2)(y - 2)}$$

$$\Rightarrow \quad \frac{dY}{dX} = \frac{(X + Y)^2}{XY} \text{ (Putting } x + 2 = X,\ y - 2 = Y)$$

$$\Rightarrow \quad dt - \frac{dt}{2t + 1} = 2\frac{dX}{X} \left(\text{putting } Y = tX \Rightarrow t + X\frac{dt}{dX} = \frac{(1 + t)^2}{t}\right)$$

$$\Rightarrow \quad \frac{y - 2}{x + 2} - \frac{1}{2}\ln\left|1 + \frac{2(y - 2)}{x + 2}\right| = 2\log(x + 2) + c$$

$\therefore$ **31. (d) 32. (b) 33. (a)**

34. (c) : $\angle IAB = \frac{\theta}{2}, \angle IAC = \frac{\theta}{2}, \frac{z_2 - z_1}{|z_2 - z_1|} = \frac{z_4 - z_1}{|z_4 - z_1|} e^{-\frac{i\theta}{2}},$

$\frac{z_3 - z_1}{|z_3 - z_1|} = \frac{z_4 - z_1}{|z_4 - z_1|} e^{-\frac{-i\theta}{2}}; \frac{(z_2 - z_1)(z_3 - z_1)}{|z_2 - z_1||z_3 - z_1|} = \frac{(z_4 - z_1)^2}{|z_4 - z_1|^2} e^0$

$$\therefore \quad \frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2} = \frac{|z_2 - z_1||z_3 - z_1|}{|z_4 - z_1|^2}$$

$$= \frac{AB \cdot AC}{(IA)^2} = \left(\frac{AB}{IA}\right)^2 \left(\frac{AC}{AB}\right)$$

35. (a) : $\frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2} = 2\left(\frac{AD}{IA}\right)^2 \left(\frac{AC}{AD}\right)$

(Since $AB = 2AD$)

$\Rightarrow (z_2 - z_1)(z_3 - z_1) = (z_4 - z_1)^2(1 + \cos\theta)\sec\theta$

36. (d)

37. (d) : Clearly, $S_n > C_n > T_n$ as 'x' is a proper fraction.

$\therefore \quad x > x^2 > x^3$ and so on.

38. (c) : We have, $S_n = C_n + T_n$

$$\Rightarrow \quad x\frac{((x^3)^{2n} - 1)}{(x^3 - 1)} = x^2 \frac{((x^3)^{2n} - 1)}{(x^3 - 1)} + x^3 \frac{((x^3)^{2n} - 1)}{(x^3 - 1)}$$

But $x \neq 1$, as $x \in \left(0, \frac{\pi}{4}\right)$, we get $x = x^2 + x^3$

$\Rightarrow x^2 + x - 1 = 0 \ (x \neq 0)$

$$\Rightarrow x = \frac{\sqrt{5} - 1}{2} = 2\sin\frac{\pi}{10} \in \left(0, \frac{\pi}{4}\right)$$

39. (A) Let $x = \sum |\vec{a} - \vec{b}|^2 = 6 - 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

Also $|\vec{a} + \vec{b} + \vec{c}|^2 \geq 0 \Rightarrow \sum \vec{a}\cdot\vec{b} \geq \frac{-3}{2}$

$\therefore$ Max$(x) = 9$

(B) $|\vec{u}| = 1, \vec{u}\cdot\vec{v}\times\vec{w} = \sqrt{59}\cos\theta$

(C) Since, the given lines intersect

$\therefore \quad (2r + 1, 3r - 1, 4r + 1) = (s + 3, 2s + k, s)$

$\Rightarrow r = \frac{-3}{2}, s = -5 \Rightarrow k = \frac{9}{2}$

Thus, **(A) → (Q) ; (B) → (P) ; (C) → (S)**

40. (A) $I = \int \frac{dx}{a^2 - x^2} = \frac{x}{a^2 - x^2} - \int \frac{2x \cdot x dx}{(a^2 - x^2)^2}$

$$= \frac{x}{a^2 - x^2} + 2\int \frac{a^2 - x^2 - a^2}{(a^2 - x^2)^2} dx$$

$$= \frac{x}{a^2 - x^2} + 2I - 2a^2 \int \frac{dx}{(a^2 - x^2)^2}$$

$$\Rightarrow \int \frac{dx}{(a^2 - x^2)^2} - \frac{1}{2a^2}\int \frac{dx}{a^2 - x^2} = \frac{x}{2a^2(a^2 - x^2)} + c$$

(B) Put $x = a\sec\theta \Rightarrow dx = a\sec\theta\tan\theta\, d\theta$

$$\therefore I = \int \frac{a\sec\theta\tan\theta d\theta}{a^2\sec^2\theta \cdot a\tan\theta} = \frac{1}{a^2}\sin\theta + c = \frac{\sqrt{x^2 - a^2}}{a^2 x} + c$$

(C) Put $x = a\sin\theta \Rightarrow dx = a\cos\theta\, d\theta$

$$\therefore I = \int \frac{a\cos\theta a\cos\theta d\theta}{a^2\sin^2\theta} = \int \cot^2\theta d\theta = -\cot\theta - \theta + c$$

$$= -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1}\frac{x}{a} + c = -\frac{\sqrt{a^2 - x^2}}{x} + \cos^{-1}\frac{x}{a} + c$$

(D) Put $x = a\sec\theta \Rightarrow dx = a\sec\theta\tan\theta\, d\theta$

$$\therefore \quad I = \int \frac{a\tan\theta a\sec\theta\tan\theta d\theta}{a\sec\theta} = a\int \tan^2\theta d\theta$$

$$= a\tan\theta - a\theta + c = \sqrt{x^2 - a^2} - a\sec^{-1}\frac{x}{a} + c$$

Thus, **(A)→(P), (B)→(R), (C)→(Q), (D)→(S)**

◆ ◆

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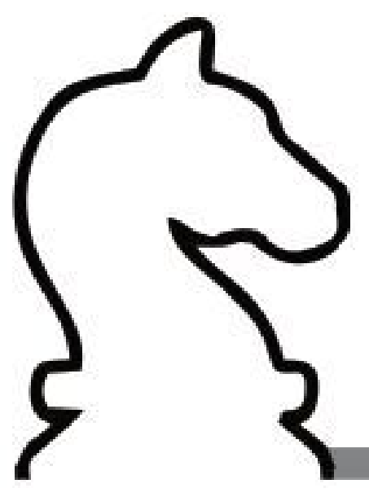
Challenging PROBLEMS ON JEE



Single Option Correct Type

1. The equation $\sin^4 x - 2\cos^2 x + a^2 = 0$ can be solved if

(a) $-\sqrt{3} \le a \le \sqrt{3}$ (b) $-\sqrt{2} \le a \le \sqrt{2}$
(c) $-1 \le a \le 1$ (d) None of these

2. Given ΔABC is inscribed in the semicircle with diameter AB. The area of ΔABC equals 2/9 of the area of the semicircle. If the measure of the smallest angle in ΔABC is x, then $\sin 2x$ is equal to

(a) $\frac{\pi}{9}$ (b) $\frac{2\pi}{9}$ (c) $\frac{\pi}{18}$ (d) $\frac{\pi}{8}$

3. Set of all the values of x satisfying the inequality

$\log_{x+\frac{1}{x}}\left(\log_2 \frac{x-1}{x+2}\right) > 0$ is

(a) $(-5, -2)$ (b) $(2, 5)$
(c) $(5, \infty)$ (d) ϕ

4. The integer value of x for which $x^2 + 19x + 92$ is square of an integer are

(a) -8 and -11 (b) -8 and 11
(c) 8 and 11 (d) None of these

5. Let a_n be the n^{th} term of the G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \ne \beta$, then the common ratio is

(a) $\frac{\alpha}{\beta}$ (b) $\frac{\beta}{\alpha}$ (c) $\sqrt{\frac{\alpha}{\beta}}$ (d) $\sqrt{\frac{\beta}{\alpha}}$

6. The coefficient of $x^n y^n$ in the expansion of $\{(1 + x)(1 + y)(x + y)\}^n$ is

(a) $\sum_{r=0}^{n} C_r^2$ (b) $\sum_{r=0}^{n} C_r^3$

(c) $\sum_{r+s=0}^{n} {}^nC_r {}^nC_s^2$ (d) None of these

7. Given 6 different toys of red colour, 5 different toys of blue colour and 4 different toys of green colour. Combination of toys that can be chosen taking at least one red and one blue toys are

(a) 31258 (b) 31248
(c) 31268 (d) None of these

8. If length of common chord of two circles $x^2 + y^2 + 8x + 1 = 0$ and $x^2 + y^2 + 2\mu y - 1 = 0$ is $2\sqrt{6}$, then value of μ are

(a) ± 2 (b) ± 3
(c) ± 4 (d) None of these

9. If two distinct chords drawn from the point (4, 4) on the parabola $y^2 = 4ax$ are bisected on the line $y = mx$, then the set of value of m is given by

(a) $\left(\frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right)$ (b) R
(c) $(0, \infty)$ (d) $(-2, 2)$

10. If the focal distance of an end of the minor axis of any ellipse (its axes as x and y axis respectively) is k and the distance between the foci is $2h$, then its equation is

(a) $\frac{x^2}{k^2} + \frac{y^2}{h^2} = 1$ (b) $\frac{x^2}{k^2} + \frac{y^2}{k^2 - h^2} = 1$

(c) $\frac{x^2}{k^2} - \frac{y^2}{k^2 - h^2} = 1$ (d) $\frac{x^2}{k^2} + \frac{y^2}{k^2 + h^2} = 1$

Comprehension Type

Paragraph for Q. No. 11 and 12

The T-polynomials are defined by $T_0(x) = 1$ and for $n \ge 1$, $\frac{dT_n}{dx} = n \cdot T_{n-1}(x)$ and $\int_0^1 T_n(x)dx = 0$

11. $T_4(x) =$

(a) $x^2(x - 1)^2 + c$ (b) $x^2(x - 1)^3 + c$
(c) $x(x - 1)^2 + c$ (d) $x^3(x - 1)^2 + c$

12. For $n \ge 2$, $T_n(1) - T_n(0) =$

(a) 0 (b) 1 (c) -1 (d) $\frac{1}{2}$

Matrix Match Type

13. Match the following.

Column-I		Column-II	
(A)	$\lim\limits_{x\to 2^+}\{x\}\dfrac{\sin(x-2)}{(x-2)^2} =$	(P)	– 3
(B)	$\lim\limits_{x\to 2}[2-x]+[x-2]-x =$	(Q)	1
(C)	$\lim\limits_{x\to 2}\{2-x\}+\{x-2\} =$	(R)	– 1
(D)	$\lim\limits_{x\to 0^+}\dfrac{e^{\frac{1}{x}}-e^{\frac{-1}{x}}}{e^{\frac{1}{x}}+e^{\frac{-1}{x}}} =$	(S)	2

(a) (A) → (Q) ; (B) → (P) ; (C) → (Q); (D) → (Q)
(b) (A) → (P) ; (B) → (R) ; (C) → (P); (D) → (Q)
(c) (A) → (P) ; (B) → (Q) ; (C) → (R); (D) → (P)
(d) (A) → (Q) ; (B) → (P) ; (C) → (R); (D) → (S)

Numerical Value Type

14. Sum of squares of all trigonometrical ratios has minimum value as ______.

15. If α_1, α_2 be the roots of $x^2 - 32x + 16 = 0$ and α_3, α_4 be the roots of $x^2 + 32x + 1 = 0$, then $\sum\limits_{i=1}^{4}\alpha_i^2 =$ ______.

16. If $f(x) = \sqrt{36-x^2} + \sin^{-1}\left(\dfrac{1}{2x}+\dfrac{x}{2}\right)$ and $g(x) = \left[\dfrac{f(x)}{k}\right]$ be continuous $\forall\, x \in R$, then least positive integral value of k is (where $[\cdot]$ denotes greatest integer function) ______.

17. Let *ABCDEFGHIJKL* be a regular do-decagon then $\dfrac{AB}{AF}+\dfrac{AF}{AB}=$ ______.

18. The number of pairs (a, b) of real numbers such that $a + \log a = b$ and $b + \log b = a$ is/are ______.

19. The value of $e^{2mi\cot^{-1}p}\cdot\left(\dfrac{pi+1}{pi-1}\right)^m$ is ______.

20. If $A=\begin{bmatrix}-1 & 1\\ 0 & -2\end{bmatrix}=B^3+C^3$, where B and C are 2×2 matrices with integer elements, then $|Tr(B) + Tr(C)| =$ ________.

SOLUTIONS

1. **(b) :** We have, $\sin^4x - 2\cos^2x + a^2 = 0$

$\Rightarrow y^2 - 2(1-y) + a^2 = 0$ [where $\sin^2x = y$]

$\Rightarrow y^2 + 2y + a^2 - 2 = 0$

For y to be real, discriminant ≥ 0

$\Rightarrow 4 - 4(a^2 - 2) \geq 0$

$\Rightarrow a^2 \leq 3$ ...(i)

Since, $\sin^2x = y \Rightarrow 0 \leq y \leq 1$

$\Rightarrow 0 \leq \sqrt{3-a^2} - 1 \leq 1$

$\Rightarrow 1 \leq \sqrt{3-a^2} \leq 2 \Rightarrow 1 \leq 3-a^2 \leq 4$

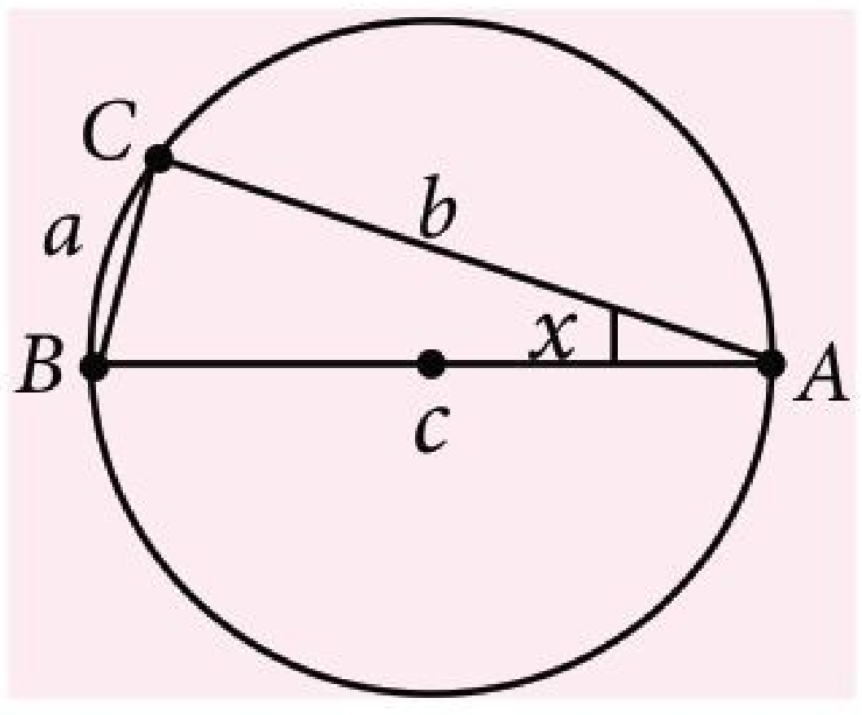
$\Rightarrow 2 - a^2 \geq 0 \Rightarrow a^2 \leq 2$

From (i) and (ii), we get $-\sqrt{2} \leq a \leq \sqrt{2}$.

2. **(a) :** Since, $a^2 + b^2 = c^2 = 4r^2$ ...(i)

Also, $\dfrac{1}{2}a\cdot b = \dfrac{2}{9}\cdot\left(\dfrac{1}{2}\pi r^2\right)$

$\Rightarrow 9ab = 2\pi r^2$ ...(ii)



From (i) and (ii), we get

$\dfrac{a^2+b^2}{9ab}=\dfrac{2}{\pi}$

$\Rightarrow \dfrac{a}{b}+\dfrac{b}{a}=\dfrac{18}{\pi}$

Now, $\angle BAC = x$, then $\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}=\dfrac{18}{\pi}$

$\Rightarrow \dfrac{\cos^2x+\sin^2x}{\sin x\cdot\cos x}=\dfrac{18}{\pi}$

$\Rightarrow \sin x\cdot\cos x=\dfrac{\pi}{18} \Rightarrow \sin 2x=\dfrac{\pi}{9}$

3. **(d) :** Since, $\log_{x+\frac{1}{x}}\log_2\left[\dfrac{x-1}{x+2}\right]>0$

$\therefore \dfrac{x-1}{x+2}>0$

$\Rightarrow x > 1, x < -2$ ...(i)

But $x+\dfrac{1}{x}>0$

$\Rightarrow \dfrac{x^2+1}{x}>0 \Rightarrow x>0$ ...(ii)

From (i) and (ii), we get

$x>1; \log_{x+\frac{1}{x}}\left(\log_2\dfrac{x-1}{x+2}\right)>0$

Taking antilog both sides, we get

$\log_2\dfrac{x-1}{x+2}>1$

Taking again antilog both sides, we get

$$\frac{x-1}{x+2} > 2 \Rightarrow \frac{x+5}{x+2} < 0$$

$\Rightarrow x \in (-5, -2)$ [But not true as $x > 1$]

4. (a) : Let $x^2 + 19x + 92 = m^2, m \in I$

$\Rightarrow x^2 + 19x + 92 - m^2 = 0$

$$\therefore \quad x = \frac{-19 \pm \sqrt{4m^2 - 7}}{2}$$

Since, $x \in I$, so $-19 \pm \sqrt{4m^2 - 7}$ must be an even integer

$\therefore \sqrt{4m^2 - 7}$ must be an odd integer

Let $\sqrt{4m^2 - 7} = 2k + 1; k \in I$

$\therefore \quad 4m^2 - (2k + 1)^2 = 7$

$\Rightarrow \quad (2m - 2k - 1)(2m + 2k + 1) = 7$

Since, 7 is prime, so possible cases are as follows:

(i) $2m - 2k - 1 = 1$ and $2m + 2k + 1 = 7$, then $m = 2$

(ii) $2m - 2k - 1 = 7$ and $2m + 2k + 1 = 1$, then $m = 2$

$\therefore$ At $m = 2, x = -8, -11$

(iii) $2m - 2k - 1 = -1$ and $2m + 2k + 1 = -7$, then $m = -2$

(iv) $2m - 2k - 1 = -7$ and $2m + 2k + 1 = -1$, then $m = -2$

$\therefore$ At $m = -2, x = -8, -11$

5. (a) : Let a G.P. with common ratio $= r$

Now, $\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + a_6 + ... + a_{200}$

$= ar + ar^3 + ar^5 + ... + ar^{199} = \alpha$ ...(i)

Also, $\sum_{n=1}^{100} a_{2n-1} = a_1 + a_3 + a_5 + ... + a_{199}$

$= a + ar^2 + ar^4 + ... + ar^{198} = \beta$ ...(ii)

On dividing (ii) by (i), we get

$$\frac{r(a + ar^2 + ar^4 + ... + ar^{198})}{(a + ar^2 + ar^4 + ... ar^{198})} = \frac{\alpha}{\beta} \Rightarrow r = \frac{\alpha}{\beta}$$

6. (b) : We have, $\{(1 + x)(1 + y)(x + y)\}^n$

$= (C_0 + C_1x + C_2x^2 + ... + C_nx^n) \times (C_0y^n + C_1y^{n-1} + ... + C_{n-1}y + C_n) \times (C_0x^n + C_1x^{n-1}y + ... + C_{n-1}xy^{n-1} + C_ny^n)$

Coefficient of $x^n y^n = C_0^3 + C_1^3 + C_2^3 + ... + C_n^3 = \sum_{r=0}^{n} C_r^3$

7. (b) : At least one red toy can be chosen in

$= {}^6C_1 + {}^6C_2 + ... + {}^6C_6 = 2^6 - 1 = 63$ ways

At least one blue toy can be chosen in

$= {}^5C_1 + {}^5C_2 + ... + {}^5C_5 = 2^5 - 1 = 31$ ways

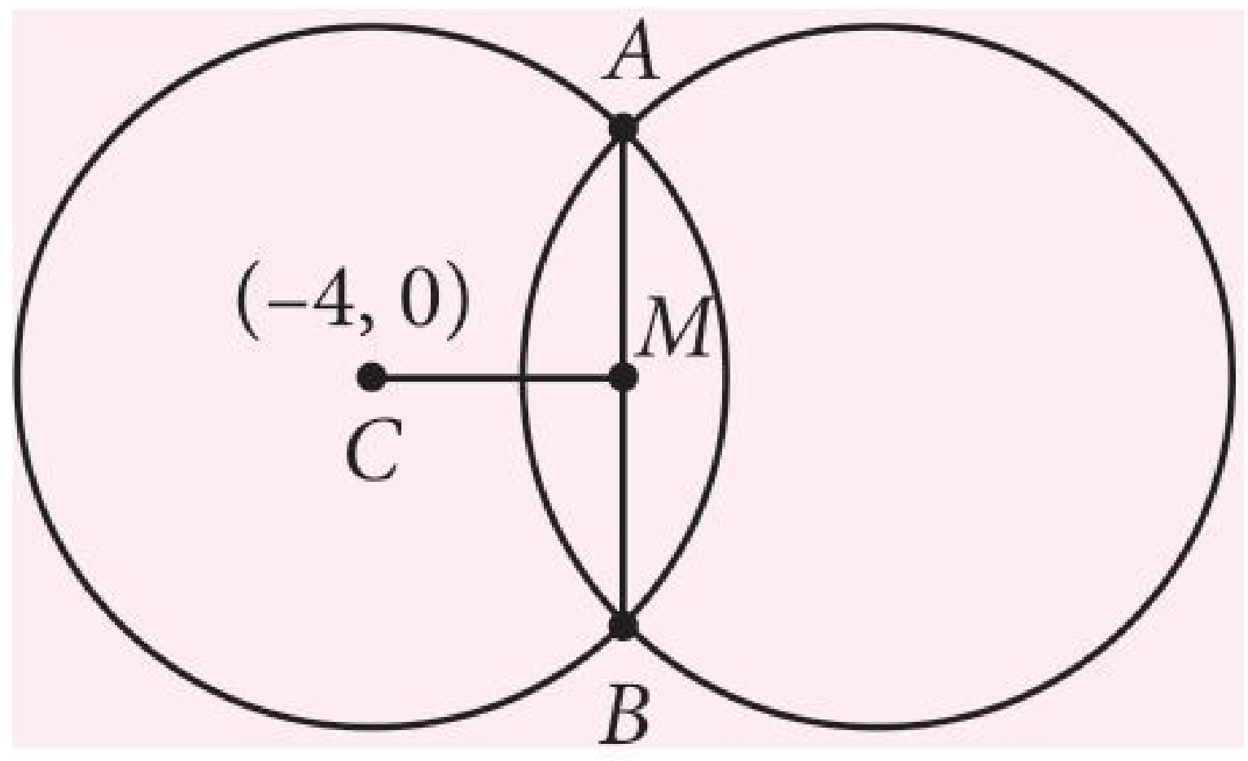
Green toys (with no restriction) can be selected in

${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 = 16$ ways

$\therefore$ Total ways of selection $= 63 \times 31 \times 16 = 31248$

8. (b) : Common chord is $S_1 - S_2 = 0$

$\Rightarrow 4x - \mu y + 1 = 0$



$\therefore \quad AC = \sqrt{15}$

$$AM = \frac{AB}{2} = \sqrt{6}$$

$\therefore \quad CM = 3$

$$\therefore \quad \text{Perpendicular distance} = \frac{|-16 + 1|}{\sqrt{16 + \mu^2}} = 3$$

$\Rightarrow \mu = \pm 3$

9. (a) : Any point on the line $y = mx$ can be taken as (t, mt).

Equation of the chord of parabola with (t, mt) as mid point $y\, mt - 2(x + t) = m^2 t^2 - 4t$

Since, it passes through (4, 4),

$\therefore \quad 4mt - 2(4 + t) = m^2t^2 - 4t$

$\Rightarrow m^2t^2 - 2(2m + 1)t + 8 = 0$

For two such chords $D > 0$

$\Rightarrow (2m + 1)^2 - 8m^2 > 0$

$$\Rightarrow 4m^2 - 4m - 1 < 0 \Rightarrow \frac{1 - \sqrt{2}}{2} < m < \frac{1 + \sqrt{2}}{2}$$

10. (b) : Let equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and e is eccentricity of ellipse.

$\therefore \quad 2h = 2ae \Rightarrow ae = h$ ...(i)

Focal distance of one end of minor axis say $(0, b)$ is k.

$\therefore \quad a + e(0) = k \Rightarrow a = k$ ...(ii)

From (i) and (ii), we get

$b^2 = a^2(1 - e^2) \Rightarrow b^2 = a^2 - a^2e^2 = k^2 - h^2$

$$\therefore \quad \text{Equation of ellipse is } \frac{x^2}{k^2} + \frac{y^2}{k^2 - h^2} = 1.$$

(11-12) : $T_1(x) = \int 1 \cdot T_0(x)\,dx = x + k$ and $\int_0^1 T_1(x)dx = 0$

gives $k = \frac{-1}{2}$. So, $T_1(x) = x - \frac{1}{2}$

Similarly, $T_2(x) = x^2 - x + \frac{1}{6}, T_3(x) = x^3 - \frac{3}{2}x^2 + \frac{x}{2}$

and $T_4(x) = x^2(x - 1)^2 + c$

$$\text{and } T_n(1) - T_n(0) = \int_0^1 \frac{dT_n(x)}{dx}\,dx = n \cdot \int_0^1 T_{n-1}(x)dx = 0$$

11. (a) **12. (a)**

13. (a) : (A) The value of $\{x\} = x - 2$ for x close to 2 and $x > 2$

$$L = \lim_{x \to 2^+} \frac{(x - 2)\sin(x - 2)}{(x - 2)^2} = \lim_{x \to 2^+} \frac{\sin(x - 2)}{x - 2} = 1.$$

(B) $\lim_{x\to 2}[2-x]+[x-2]-x$

$= \lim_{x\to 2} 2+[-x]+[x]-2-x = \lim_{x\to 2}[-x]+[x]-x$

R.H.L. $= \lim_{x\to 2^+}[-x]+[x]-x = -3+2-2=-3$

L.H.L. $= \lim_{x\to 2^-}[-x]+[x]-x = -2+1-2=-3$

$\therefore\ L=-3.$

(C) $\{2-x\}+\{x-2\}=2-x-[2-x]+x-2-[x-2]$

$=-[2-x]-[x-2]=-2-[-x]-[x]+2=-[-x]-[x]$

R.H.L.$= \lim_{x\to 2^+}-[-x]-[x]=3-2=1$

L.H.L.$= \lim_{x\to 2^-}-[-x]-[x]=2-1=1$

$\therefore$ Limit = 1.

(D) R.H.L. $= \lim_{x\to 0^+}\dfrac{e^{\frac{1}{x}}-e^{\frac{-1}{x}}}{e^{\frac{1}{x}}+e^{\frac{-1}{x}}} = \lim_{x\to 0}\dfrac{1-e^{\frac{-2}{x}}}{1+e^{\frac{-2}{x}}} = \dfrac{1-0}{1+0}=1$

14. (7) : $\sin^2\theta+\cos^2\theta+\sec^2\theta+\text{cosec}^2\theta+\tan^2\theta+\cot^2\theta$

$=1+(1+\tan^2\theta)+(1+\cot^2\theta)+\tan^2\theta+\cot^2\theta$

$=3+2(\tan^2\theta+\cot^2\theta) = 3+4\left(\dfrac{\tan^2\theta+\cot^2\theta}{2}\right)$

$\geq 3+4\sqrt{\tan^2\theta\cot^2\theta}$ [$\because$ A.M. $\geq$ G.M.]

≥ 7

$\therefore$ Minimum value = 7.

15. (2014) : $\alpha_1+\alpha_2=32,\ \alpha_1\alpha_2=16$

$\alpha_3+\alpha_4=-32\ ;\ \alpha_3\alpha_4=1$

$\therefore\ \sum_{i=1}^{4}\alpha_i^2=\alpha_1^2+\alpha_2^2+\alpha_3^2+\alpha_4^2$

$=(\alpha_1+\alpha_2)^2-2\alpha_1\alpha_2+(\alpha_3+\alpha_4)^2-2\alpha_3\alpha_4$

$=32^2-2(16)+(-32)^2-2\cdot 1$

$=2(1024-16-1)=2014$

16. (8) : $\sqrt{36-x^2}$ is defined if $x^2\leq 36$

$\Rightarrow -6\leq x\leq 6$

and, $\sin^{-1}\left(\dfrac{1}{2x}+\dfrac{x}{2}\right)=\sin^{-1}\left(\dfrac{1+x^2}{2x}\right)$ is defined if

$\left|\dfrac{1+x^2}{2x}\right|\leq 1$ [taking $x\neq 0$]

i.e., if $\dfrac{1+x^2}{2|x|}\leq 1 \Rightarrow 1+x^2\leq 2|x|$

$$\Rightarrow \begin{cases}1+x^2-2x\leq 0, & \text{if } x>0\\ 1+x^2+2x\leq 0, & \text{if } x<0\end{cases}$$

$$\Rightarrow \begin{cases}(1-x)^2\leq 0, & \text{if } x>0\\ (1+x)^2\leq 0, & \text{if } x<0\end{cases} \Rightarrow x=\pm 1$$

$\Rightarrow D_f=\{-1,1\}\ \therefore\ f(-1)=\sqrt{35}-\dfrac{\pi}{2}=4.35$ (approx.)

$f(1)=\sqrt{35}+\dfrac{\pi}{2}=7.49$ (approx.)

$\therefore\ R_f=\{4.35, 7.49\}$ (approx.)

Now, $g(x)=\left[\dfrac{f(x)}{k}\right]$ will be continuous $\forall\ x\in R$

if $\dfrac{f(x)}{k}\in(0,1)$ for which value of k must exceed the greatest value of $f(x)$ *i.e.*, 7.49

$\Rightarrow$ least positive integral value of $c=8$.

17. (4) : Let R be the circumradius, then

$AB=2R\sin\left(\dfrac{\pi}{12}\right)$ and $AF=2R\sin\left(\dfrac{5\pi}{12}\right)$

So, the required quantity is $\dfrac{2R\sin\theta}{2R\sin 5\theta}+\dfrac{2R\sin 5\theta}{2R\sin\theta}$.

$\left[\text{where } \theta=\dfrac{\pi}{12}\right]$

$=\dfrac{\sin^2\theta+\sin^2 5\theta}{\sin 5\theta\sin\theta}=\dfrac{1-\cos 2\theta+1-\cos 10\theta}{\cos 4\theta-\cos 6\theta}$

= 4 (on simplification)

18. (1) : Simplify to get, $\log a=-a+\dfrac{1}{a}$

If $a>1$, then L.H.S. >0 and R.H.S. <0.

So, no solution.

If $a<1$, then L.H.S. <0 and R.H.S. >0.

So no solution in this case also.

So, only possible value of $a=1$.

Hence only 1 pair exists, *i.e.*, $(a,b)=(1,1)$.

19. (1) : Let $\cot^{-1}p=\theta$, then $\cot\theta=p$

Now, $e^{2mi\theta}.\left(\dfrac{i\cot\theta+1}{i\cot\theta-1}\right)^m=e^{2mi\theta}\left[\dfrac{i(\cot\theta-i)}{i(\cot\theta+i)}\right]^m$

$=e^{2mi\theta}\left[\dfrac{\cos\theta-i\sin\theta}{\cos\theta+i\sin\theta}\right]^m$

$=e^{2mi\theta}\left(\dfrac{e^{-i\theta}}{e^{i\theta}}\right)^m=e^{2mi\theta}(e^{-2i\theta})^m=e^{2mi\theta}e^{-2mi\theta}=e^0=1$

20. (3) : We have, $A^2+3A+2I=O$

$\Rightarrow\ A=(A+I)^3+(-I)^3$

i.e., $B=A+I$ and $C=-I$

i.e., $B=\begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}$ and $C=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

So, $Tr(B)+Tr(C)=-1-2=-3$

◆ ◆

beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. If a, b and c are three distinct real numbers, which are in G.P. and $a + b + c = xb$. Then
(a) $x < -1$ or $x > 3$ (b) $-1 < x < 3$
(c) $-1 < x < 2$ (d) $0 < x < 1$

2. If three unequal numbers a, b and c are in H.P. and their squares are in A.P., then $a : b : c =$
(a) $1+\sqrt{3} : -2 : 1-\sqrt{3}$ (b) $1 : 2 : 3$
(c) $\sqrt{3}+1 : \sqrt{3}-1 : 2$ (d) $3 : 2 : 1$

3. If $f(x) = \cos[\pi]x + \cos[\pi x]$, where $[x]$ is the greatest integer function of x, then $f\left(\frac{\pi}{2}\right) =$
(a) 0 (b) cos 3
(c) cos 4 (d) none of these

4. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on a
(a) straight line (b) circle
(c) ellipse (d) none of these

5. The coordinates of the point on the y-axis which is equidistant from the points $(-5, 4)$ and $(3, -2)$ are
(a) $\left(0, \frac{4}{3}\right)$ (b) $\left(0, \frac{3}{4}\right)$
(c) $\left(0, \frac{7}{3}\right)$ (d) $(0, 1)$

6. The area of the triangle with vertices $(p + 1, 1)$ $(2p + 1, 3)$ and $(2p + 2, 2p)$ is 9 square units. Then the integral value of p is equal to
(a) 4 (b) 1
(c) 2 (d) 3

7. Let z_1, z_2 and z_3 be three points on $|z| = 1$ and $z_1 + z_2 + z_3 = 0$. If θ_1, θ_2 and θ_3 be the arguments of z_1, z_2 and z_3 respectively, then $\cos(\theta_1 - \theta_2) + \cos(\theta_2 - \theta_3) + \cos(\theta_3 - \theta_1)$ is equal to
(a) 0 (b) $-\frac{3}{2}$ (c) -1 (d) $\frac{3}{2}$

8. If $x = a + bi$ is a complex number such that $x^2 = 3 + 4i$ and $x^3 = 2 + 11i$ where $i = \sqrt{-1}$, then $(a + b)$ is equal to
(a) 2 (b) 3 (c) 4 (d) 5

9. $\lim\limits_{x \to 0} \dfrac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x} =$
(a) 1 (b) 2 (c) $\log_e a$ (d) $\log_a e$

10. If $f(x) = \frac{1}{3}\left(f(x+1) + \frac{5}{f(x+2)}\right)$ and $f(x) > 0$, $\forall\, x \in R$, then $\lim\limits_{x \to \infty} f(x)$ is
(a) 0 (b) $\sqrt{\frac{2}{5}}$ (c) $\sqrt{\frac{5}{2}}$ (d) ∞

SECTION-II

Numerical Answer Type

11. If the 5th term of a G.P. is 2 and the product of 9 terms of the progression is $128k$, then the value of k is ____.

12. If $A + B = 225°$, then $\dfrac{1+\cot A}{\cot A} \cdot \dfrac{1+\cot B}{\cot B} =$ ____.

13. If $A(0, \sqrt{2})$ and $B(2\sqrt{2}, 0)$ are two vertices of ΔABC and $AC = BC$ and if the equation of BC is $x = 2\sqrt{2}$, then the area of the triangle is ____ (sq.units).

14. If $y = \sqrt{2x - x^2}$ and k is a numerical quantity such that $y^3 \dfrac{d^2y}{dx^2} + k = 0$, then k should be ____.

15. If the function $y = f(x)$ is given by $x = 3t^2$, $y = 3t - t^3$ satisfies $k\dfrac{d^2y}{dx^2}(y-\sqrt{3x}) = x+3$. Then the numerical quantity $\sqrt{k}$ should be ____.

SOLUTIONS

1. (a): Given, $a + b + c = xb$

$\Rightarrow a + ar + ar^2 = x \cdot ar$

[let the common ratio of the G.P. be r]

$\Rightarrow 1 + (1 - x)r + r^2 = 0$

$\because$ r is real $\therefore (1 - x)^2 - 4 \cdot 1 \cdot 1 > 0$

$\Rightarrow x^2 - 2x + 1 - 4 > 0 \Rightarrow x^2 - 2x - 3 > 0$

$\Rightarrow (x + 1)(x - 3) > 0 \Rightarrow x < -1$ or $x > 3$

2. (a): Let $a = \dfrac{1}{k-d}$, $b = \dfrac{1}{k}$ and $c = \dfrac{1}{k+d}$

So, that a, b, c are in H.P.

Also given that a^2, b^2, c^2 are in A.P.

$$\therefore \frac{1}{(k-d)^2} + \frac{1}{(k+d)^2} = \frac{2}{k^2} \Rightarrow \frac{2(k^2+d^2)}{(k^2-d^2)^2} = \frac{2}{k^2}$$

$\Rightarrow k^2(k^2 + d^2) = k^4 - 2k^2d^2 + d^4$

$\Rightarrow k^4 + k^2d^2 = k^4 - 2k^2d^2 + d^4 \Rightarrow d^4 - 3k^2d^2 = 0$

$\Rightarrow d^2(d^2 - 3k^2) = 0, \because d \neq 0 \therefore d^2 - 3k^2 = 0$

$\Rightarrow d^2 = 3k^2 \Rightarrow d = \pm\sqrt{3}k$

$$\therefore a = \frac{1}{k-\sqrt{3}k} \text{ or } \frac{1}{k+\sqrt{3}k},\ b = \frac{1}{k} \text{ and}$$

$$c = \frac{1}{k+\sqrt{3}k} \text{ or } \frac{1}{k-\sqrt{3}k}$$

$$\therefore a:b:c = \frac{1}{k-\sqrt{3}k} : \frac{1}{k} : \frac{1}{k+\sqrt{3}k}$$

$$= \frac{1}{1-\sqrt{3}} : 1 : \frac{1}{1+\sqrt{3}} = \frac{1+\sqrt{3}}{-2} : 1 : \frac{1-\sqrt{3}}{-2}$$

$$= (1+\sqrt{3}) : -2 : (1-\sqrt{3})$$

$$\text{or, } a:b:c = \frac{1}{k+\sqrt{3}k} : \frac{1}{k} : \frac{1}{k-\sqrt{3}k}$$

$$= 1-\sqrt{3} : -2 : 1+\sqrt{3}$$

3. (c): $f(x) = \cos[\pi]x + \cos[\pi x]$

$$\therefore f\left(\frac{\pi}{2}\right) = \cos[\pi]\frac{\pi}{2} + \cos\left[\frac{\pi^2}{2}\right]$$

$$= \cos\frac{3\pi}{2} + \cos 4 \qquad [\because \pi = 3.14 \ \therefore [\pi] = 3]$$

$= 0 + \cos 4$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \cos 4$$

4. (a): Let $x_1 = a$, $x_2 = ar$ and $x_3 = ar^2$

and $y_1 = b$, $y_2 = br$ and $y_3 = br^2$

$\therefore$ The vertices of the triangle are $A(a, b)$ $B(ar, br)$ and $C(ar^2, br^2)$

$$\text{Now slope of } AB = \frac{br-b}{ar-a} = \frac{b}{a}$$

$$\text{and slope of } BC = \frac{br^2-br}{ar^2-ar} = \frac{b}{a}$$

Hence A, B, C are collinear.

5. (c): Let the point on y-axis be $(0, k)$.

Now $(0 + 5)^2 + (k - 4)^2 = (0 - 3)^2 + (k + 2)^2$

$\Rightarrow 25 + k^2 - 8k + 16 = 9 + k^2 + 4k + 4$

$$\Rightarrow 12k = 28 \Rightarrow k = \frac{7}{3}$$

$$\therefore \text{ The required point is } \left(0, \frac{7}{3}\right)$$

6. (a): $$\frac{1}{2}\left|\{(p+1)(3-2p) + (2p+1)(2p-1) + (2p+2)(1-3)\}\right| = 9$$

$$\Rightarrow \left|3p - 2p^2 + 3 - 2p + 4p^2 - 1 - 4p - 4\right| = 18$$

$$\Rightarrow \left|2p^2 - 3p - 2\right| = 18$$

$$\Rightarrow 2p^2 - 3p - 2 = 18 \quad \text{or} \quad 2p^2 - 3p - 2 = -18$$

when $2p^2 - 3p - 20 = 0$, then $(2p + 5)(p - 4) = 0$

$$\therefore p = 4, \frac{-5}{2}$$

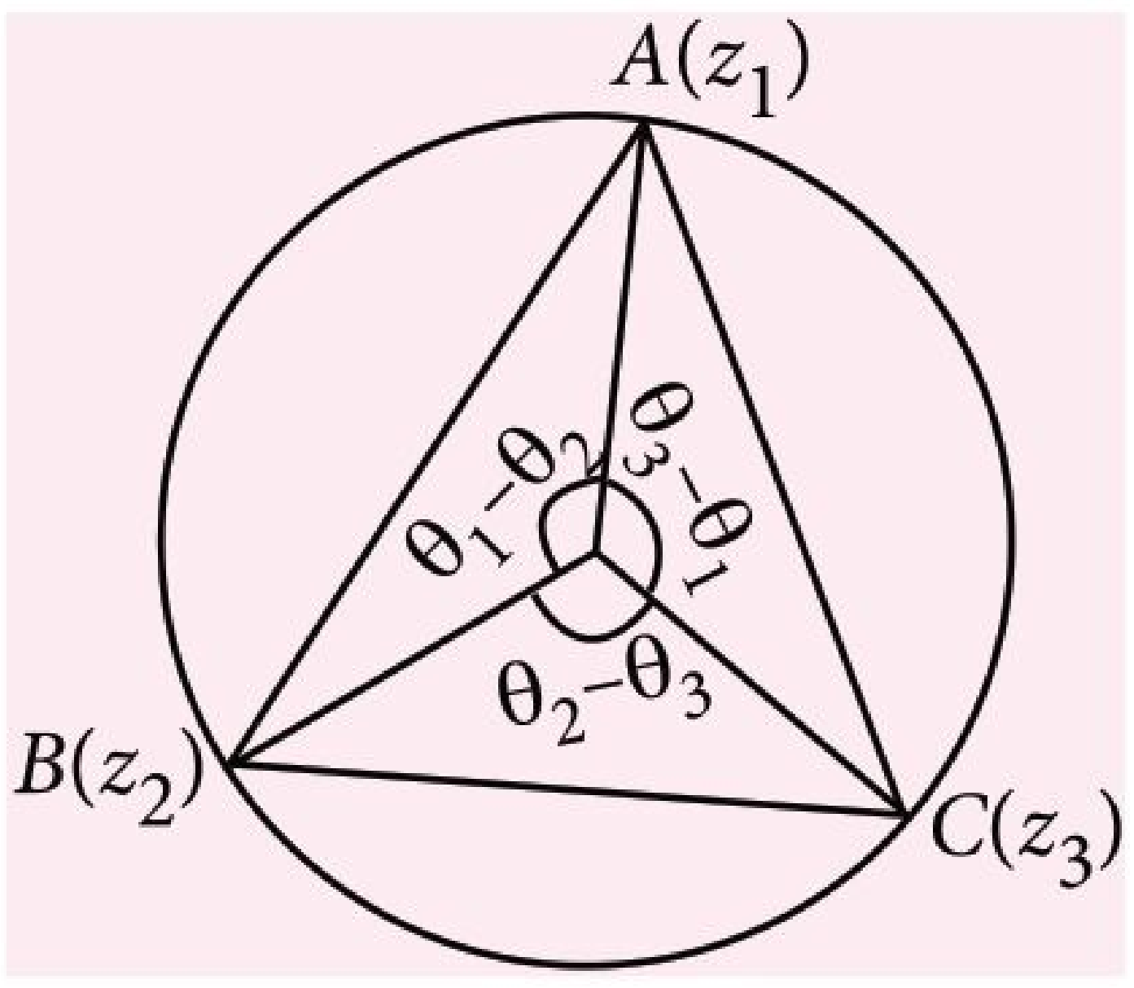
Also when $2p^2 - 3p + 16 = 0$. Then there is no real value of p.

7. (b): Since $|z_1| = |z_2| = |z_3| = 1$ and $z_1 + z_2 + z_3 = 0$

$\therefore$ Triangle ABC is an equilateral triangle

$\therefore \theta_1 - \theta_2 = \theta_2 - \theta_3$

$$= \theta_3 - \theta_1 = \frac{2\pi}{3}$$



$$\therefore \cos(\theta_1 - \theta_2) + \cos(\theta_2 - \theta_3) + \cos(\theta_3 - \theta_1) = -\frac{3}{2}$$

8. (b): $$x = \frac{x^3}{x^2} = \frac{2+11i}{3+4i} \times \frac{3-4i}{3-4i}$$

$$= \frac{6+(33-8)i-44i^2}{9-16i^2} = \frac{(6+44)+25i}{9+16} = \frac{50+25i}{25} = 2+i$$

So, $a + b = 2 + 1 = 3$

9. (c) : $\lim\limits_{x\to 0}\dfrac{a^{\tan x}-a^{\sin x}}{\tan x-\sin x}$

$$=\lim_{x\to 0}\frac{\dfrac{a^{\tan x}-1}{\tan x}\tan x-\dfrac{a^{\sin x}-1}{\sin x}\sin x}{\tan x-\sin x}$$

$$=\lim_{x\to 0}\frac{\log_e a\tan x-\log_e a\sin x}{\tan x-\sin x}=\log_e a$$

10. (c) : $\lim\limits_{x\to\infty}f(x)=\lim\limits_{x\to\infty}\dfrac{1}{3}\left(f(x+1)+\dfrac{5}{f(x+2)}\right)$

$$=\frac{1}{3}\left(f(\infty+1)+\frac{5}{f(\infty+2)}\right)=\frac{1}{3}\left(f(\infty)+\frac{5}{f(\infty)}\right)$$

$\Rightarrow\ K=\dfrac{1}{3}\left(K+\dfrac{5}{K}\right)$

$\Rightarrow\ 3K^2=K^2+5\ \Rightarrow\ 2K^2=5\ \Rightarrow\ K^2=\dfrac{5}{2}$

$\Rightarrow\ K=\pm\sqrt{\dfrac{5}{2}}\ \Rightarrow\ \lim\limits_{x\to\infty}f(x)=\sqrt{\dfrac{5}{2}}$ $\quad[\because f(x)>0]$

11. (4) : Given, $ar^4=2$ ...(i)

Also, $a\cdot ar\cdot ar^2 \dots ar^8=128\,k$ ...(ii)

$\Rightarrow\ a^9\cdot r^{1+2+\dots+8}=128\,k\Rightarrow a^9\cdot r^{4\times 9}=128\,k$

$\Rightarrow\ (ar^4)^9=128\,k\ \Rightarrow\ 2^9=128\,k$ (by (i))

$\Rightarrow\ 128\times 4=128\,k\ \Rightarrow\ k=4$

12. (2) : If $A+B=225^\circ$, then $\cot(A+B)=\cot 225^\circ$

$$\Rightarrow\ \frac{\cot A\cot B-1}{\cot A+\cot B}=\cot 225^\circ=\cot(180^\circ+45^\circ)=\cot 45^\circ=1$$

$\Rightarrow\ \cot A\cot B-1=\cot A+\cot B$

$\Rightarrow\ \cot A\cot B=1+\cot A+\cot B$

$\Rightarrow\ 2\cot A\cot B=1+\cot A+\cot B+\cot A\cot B$

$\Rightarrow\ 2\cot A\cot B=(1+\cot A)(1+\cot B)$

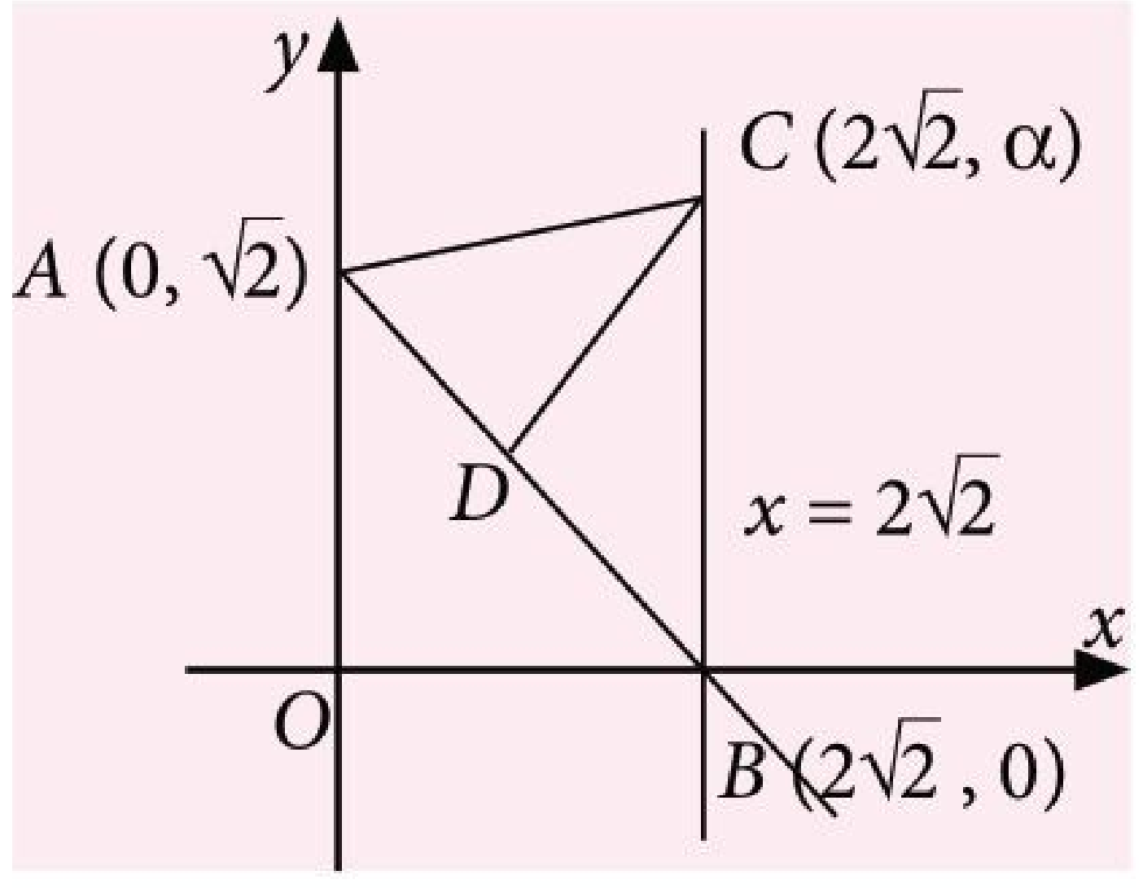
$\therefore\ \dfrac{1+\cot A}{\cot A}\cdot\dfrac{1+\cot B}{\cot B}=2$

13. (5) : Let $A\equiv(0,\sqrt{2})$, $B\equiv(2\sqrt{2},0)$

and third vertex be $C\equiv(2\sqrt{2},\alpha)$

Now $AB=\sqrt{(2\sqrt{2}-0)^2+(0-\sqrt{2})^2}$

$AB=\sqrt{10}$



Now $AC=BC$

$\Rightarrow\ (2\sqrt{2})^2+(\alpha-\sqrt{2})^2=\alpha^2$

$\Rightarrow\ 8+(\alpha-\sqrt{2})^2=\alpha^2\ \Rightarrow\ \alpha=\dfrac{5}{\sqrt{2}}$

Equation of AB is $\dfrac{x}{2\sqrt{2}}+\dfrac{y}{\sqrt{2}}=1$

$\Rightarrow\ x+2y=2\sqrt{2}$

Now $CD=\dfrac{2\sqrt{2}+2\times\dfrac{5}{\sqrt{2}}-2\sqrt{2}}{\sqrt{1^2+2^2}}=\dfrac{\sqrt{2}\times 5}{\sqrt{5}}=\sqrt{10}$

$\therefore$ Area of $\Delta ABC=\dfrac{1}{2}AB\times CD$

$=\dfrac{1}{2}\times\sqrt{10}\times\sqrt{10}=\dfrac{1}{2}\times 10=5$ sq. units

14. (1) : $\dfrac{dy}{dx}=\dfrac{2-2x}{2\sqrt{2x-x^2}}=\dfrac{1-x}{y}$ ...(i)

$\Rightarrow\ y\dfrac{dy}{dx}=1-x$

Differentiating again, we get

$$\left(\frac{dy}{dx}\right)^2+y\frac{d^2y}{dx^2}=-1$$

$\Rightarrow\ y^3\dfrac{d^2y}{dx^2}=-y^2-(1-x)^2$

$=-2x+x^2-(1-x)^2=-1$ [From (i)]

$\therefore\ k=1$

15. (6) : Given, $y=3t-t^3$ ...(i)

and $x=3t^2$ ...(ii)

$\dfrac{dy}{dx}=\dfrac{1-t^2}{2t}$

$\dfrac{d^2y}{dx^2}=\dfrac{(-2t)(2t)-2(1-t^2)}{4t^2}\dfrac{dt}{dx}=\dfrac{-1-t^2}{2t^2}\cdot\dfrac{1}{6t}$

$\Rightarrow\ -12\dfrac{d^2y}{dx^2}t^3=1+t^2\ \Rightarrow\ -36\dfrac{d^2y}{dx^2}t^3=3+3t^2$

$\Rightarrow\ 36\dfrac{d^2y}{dx^2}(y-\sqrt{3x})=x+3$ [Using (i) and (ii)]

$\therefore\ k=36\ \Rightarrow\ \sqrt{k}=6$

⁂

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(c)	**2.**	(b)	**3.**	(a)	**4.**	(b)	**5.**	(c)
6.	(b)	**7.**	(c,d)	**8.**	(a,c)	**9.**	(a,b,c)	**10.**	(a,b,c)
11.	(a,b,c)	**12.**	(a,c)	**13.**	(b)	**14.**	(c)	**15.**	(b)
16.	(b)	**17.**	(1)	**18.**	(3)	**19.**	(1526)	**20.**	(9)

CONCEPT
MAP

BINOMIAL THEOREM

Class XI

Expansions

For positive integral index ‘n’, $n \in N$

- $(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + \ldots + {}^nC_na^n$
- $(a+x)^n = a^n\left(1+\dfrac{x}{a}\right)^n, \left|\dfrac{x}{a}\right| < 1$
- $(1+x)^n = \displaystyle\sum_{r=0}^{n} {}^nC_rx^r$
- $(1-x)^n = \displaystyle\sum_{r=0}^{n} (-1)^r\,{}^nC_rx^r$

For negative integral index ‘$-n$’, $n \in N$

- $(1+x)^{-n} = \displaystyle\sum_{r=0}^{n} (-1)^n\,{}^nC_rx^r, (1-x)^{-n} = \sum_{r=0}^{n} {}^nC_rx^r$

General Term

- $T_{r+1} = {}^nC_r\,x^{n-r}\,a^r$
- T_{r+1} from the end $= T_{n-r+1}$ from the beginning

Middle Terms

- If n is even, $T_{\frac{n}{2}+1}$ is the middle term.
- If n is odd, $T_{\frac{n+1}{2}}, T_{\frac{n+3}{2}}$ are two middle terms.

Multinomial Theorem

- $(x_1+x_2+x_3+\ldots+x_r)^n = \displaystyle\sum \frac{n!}{p_1!\,p_2!\ldots p_r!}x_1^{p_1}x_2^{p_2}\ldots x_r^{p_r}$

Number of Terms in the Expansion

- Number of terms in the expansion of $(x+y+z)^n$ $= x^n + {}^nC_1x^{n-1}(y+z) + {}^nC_2x^{n-2}(y+z)^2 + \ldots + {}^nC_n(y+z)^n$ $= 1 + 2 + 3 + \ldots + (n+1) = \displaystyle\sum_{r=1}^{n+1} r = \frac{1}{2}(n+1)(n+2)$
- The number of terms in the expansion of $(x_1 + x_2 + x_3 + \ldots + x_k)^n$ is equal to the number of non-negative integral solutions of the equation $r_1 + r_2 + \ldots + r_k = n$
  The number of such solutions $= {}^{n+k-1}C_{k-1}$.

Binomial Theorem for Any Index

- $(1+x)^n = 1 + nx + \dfrac{n(n-1)x^2}{2!} + \ldots + \dfrac{n(n-1)\ldots(n-r+1)x^r}{r!} + \ldots\infty, |x| < 1$
- $T_{r+1} = \dfrac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r$
- T_{r+1} in $(1+x)^{-n} = \dfrac{(-1)^r n(n+1)(n+2)\ldots(n+r-1)}{r!}x^r$
- T_{r+1} in $(1-x)^{-n} = \dfrac{n(n+1)(n+2)\ldots(n+r-1)}{r!}x^r$

Binomial Coefficients

Greatest Binomial Coefficient is

- ${}^nC_{\frac{n}{2}}$, if n is even
- ${}^nC_{\frac{n-1}{2}} = {}^nC_{\frac{n+1}{2}}$, if n is odd

Some Important Results

- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_r = \dfrac{n}{r}\,{}^{n-1}C_{r-1} = \dfrac{n}{r}\left(\dfrac{n-1}{r-1}\right){}^{n-2}C_{r-2}$
- ${}^nC_x = {}^nC_y \Rightarrow$ Either $x = y$ or $x + y = n$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- $\displaystyle\sum_{r=0}^{n} {}^nC_r = \sum_{r=0}^{n} C_r = 2^n$
- $C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots = 2^{n-1}$
- $\displaystyle\sum_{r=0}^{n} r \cdot C_r = n2^{n-1}$
- $C_0^2 - C_1^2 + C_2^2 - \ldots + (-1)^nC_n^2 = \begin{cases} 0, n \text{ is odd} \\ (-1)^{\frac{n}{2}}\,{}^nC_{\frac{n}{2}}, n \text{ is even} \end{cases}$
- $C_0 - C_2 + C_4 - C_6 + \ldots = 2^{\frac{n}{2}}\cos\dfrac{n\pi}{4}$
- $C_1 - C_3 + C_5 - C_7 + \ldots = 2^{\frac{n}{2}}\sin\dfrac{n\pi}{4}$
- If $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \ldots p_r^{\alpha_r}$, where $p_1, p_2, \ldots, p_r$ are prime numbers and $\alpha_1, \alpha_2, \ldots, \alpha_r$ are positive integers, then sum of positive divisors of $n = (1 + p_1 + p_1^2 + \ldots + p_1^{\alpha_1})$ $(1 + p_2 + p_2^2 + \ldots + p_2^{\alpha_2}) \ldots (1 + p_r + p_r^2 + \ldots + p_r^{\alpha_r})$.

INVERSE TRIGONOMETRIC FUNCTIONS

CONCEPT MAP

Class XII

Function	Domain	Range
$\sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$(-\pi/2, \pi/2)$
$\cot^{-1} x$	R	$(0, \pi)$
$\text{cosec}^{-1} x$	$R - (-1, 1)$	$[-\pi/2, \pi/2] - \{0\}$
$\sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \{\pi/2\}$

- $\sin(\sin^{-1} x) = x \ \forall\ x \in [-1, 1]$
- $\cos(\cos^{-1} x) = x \ \forall\ x \in [-1, 1]$
- $\tan(\tan^{-1} x) = x \ \forall\ x \in R$
- $\cot(\cot^{-1} x) = x \ \forall\ x \in R$
- $\text{cosec}(\text{cosec}^{-1} x) = x \ \forall\ x \in R - (-1, 1)$
- $\sec(\sec^{-1} x) = x \ \forall\ x \in R - (-1, 1)$

- $\sin^{-1}(-x) = -\sin^{-1} x, x \in [-1, 1]$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x, x \in [-1, 1]$
- $\tan^{-1}(-x) = -\tan^{-1} x, x \in R$
- $\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in R$
- $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x, |x| \geq 1$
- $\sec^{-1}(-x) = \pi - \sec^{-1} x, |x| \geq 1$

- $\sin^{-1} x + \cos^{-1} x = \pi/2$
- $\tan^{-1} x + \cot^{-1} x = \pi/2$
- $\text{cosec}^{-1} x + \sec^{-1} x = \pi/2$

- $\sin^{-1} x \pm \sin^{-1} y$
  $$= \begin{cases} \sin^{-1}(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}) \ \forall\ x, y \geq 0, x^2 + y^2 \leq 1 \\ \pi - \sin^{-1}(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}) \ \forall\ x, y \geq 0, x^2 + y^2 > 1 \end{cases}$$
- $\cos^{-1} x \pm \cos^{-1} y$
  $$= \begin{cases} \cos^{-1}(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}) \ \forall\ x, y \geq 0, x^2 + y^2 \leq 1 \\ \pi - \cos^{-1}(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}) \forall\ x, y \geq 0, x^2 + y^2 > 1 \end{cases}$$
- $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right), xy < 1$
- $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right), xy > -1$

- $\sin^{-1}(1/x) = \text{cosec}^{-1} x; x \geq 1 \text{ or } x \leq -1$
- $\cos^{-1}(1/x) = \sec^{-1} x; x \geq 1 \text{ or } x \leq -1$
- $\tan^{-1}(1/x) = \cot^{-1} x; x > 0$

- $$3\sin^{-1} x = \begin{cases} \sin^{-1}(3x - 4x^3), \text{ if } -\dfrac{1}{2} \leq x \leq \dfrac{1}{2} \\ \pi - \sin^{-1}(3x - 4x^3), \text{ if } \dfrac{1}{2} < x \leq 1 \\ -\pi - \sin^{-1}(3x - 4x^3), \text{ if } -1 \leq x < -\dfrac{1}{2} \end{cases}$$
- $$3\cos^{-1} x = \begin{cases} \cos^{-1}(4x^3 - 3x), \text{ if } \dfrac{1}{2} \leq x \leq 1 \\ 2\pi - \cos^{-1}(4x^3 - 3x), \text{ if } -\dfrac{1}{2} \leq x \leq \dfrac{1}{2} \\ 2\pi + \cos^{-1}(4x^3 - 3x), \text{ if } -1 \leq x \leq -\dfrac{1}{2} \end{cases}$$
- $$3\tan^{-1} x = \begin{cases} \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right), \text{ if } -\dfrac{1}{\sqrt{3}} < x < \dfrac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right), \text{ if } x > \dfrac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right), \text{ if } x < -\dfrac{1}{\sqrt{3}} \end{cases}$$

- $$2\sin^{-1} x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}), |x| \leq \dfrac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}), \dfrac{1}{\sqrt{2}} \leq x \leq 1 \end{cases}$$
- $$2\cos^{-1} x = \begin{cases} \cos^{-1}(2x^2 - 1), 0 \leq x \leq 1 \\ 2\pi - \cos^{-1}(2x^2 - 1), -1 \leq x \leq 0 \end{cases}$$
- $$2\tan^{-1} x = \begin{cases} \tan^{-1}\left(\dfrac{2x}{1-x^2}\right), -1 < x < 1 \\ \sin^{-1}\left(\dfrac{2x}{1+x^2}\right), |x| \leq 1 \\ \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right), x \geq 0 \end{cases}$$

- $\dfrac{1}{2}\sin^{-1} x = \tan^{-1}\left(\dfrac{1-\sqrt{1-x^2}}{x}\right)$
- $\dfrac{1}{2}\cos^{-1} x = \tan^{-1}\left(\dfrac{1-x}{\sqrt{1-x^2}}\right)$
- $\dfrac{1}{2}\tan^{-1} x = \tan^{-1}\left(\dfrac{\sqrt{x^2+1}-1}{x}\right)$

OLYMPIAD CORNER

1. Let ABC be an acute triangle with D, E and F the feet of the altitudes lying on BC, CA and AB respectively. One of the intersection points of the line EF and the circumcircle is P. The lines BP and DF meet at point Q. Prove that $AP = AQ$.

2. Let $\mathbb{N}$ be the set of all positive integers. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that the number $(f(m) + n)(m + f(n))$ is a square for all $m, n \in \mathbb{N}$.

3. A sequence $x_1, x_2, \ldots$ is defined by $x_1 = 1$ and $x_{2k} = -x_k$, $x_{2k-1} = (-1)^{k+1}x_k$ for all $k \geq 1$. Prove that $x_1 + x_2 + \ldots + x_n \geq 0$ for $n \geq 1$.

4. Find the least positive integer n for which there exists a set $\{s_1, s_2, \ldots, s_n\}$ consisting of n distinct positive integers such that $\left(1-\frac{1}{s_1}\right)\left(1-\frac{1}{s_2}\right)\ldots\left(1-\frac{1}{s_n}\right)=\frac{51}{2010}$.

5. At Matilda's birthday party, the ratio of people who ate ice cream to people who ate cake was 3 : 2. People who ate both ice cream and cake were included in both categories. If 120 people were at the party, what is the maximum number of people who could have eaten both ice cream and cake?

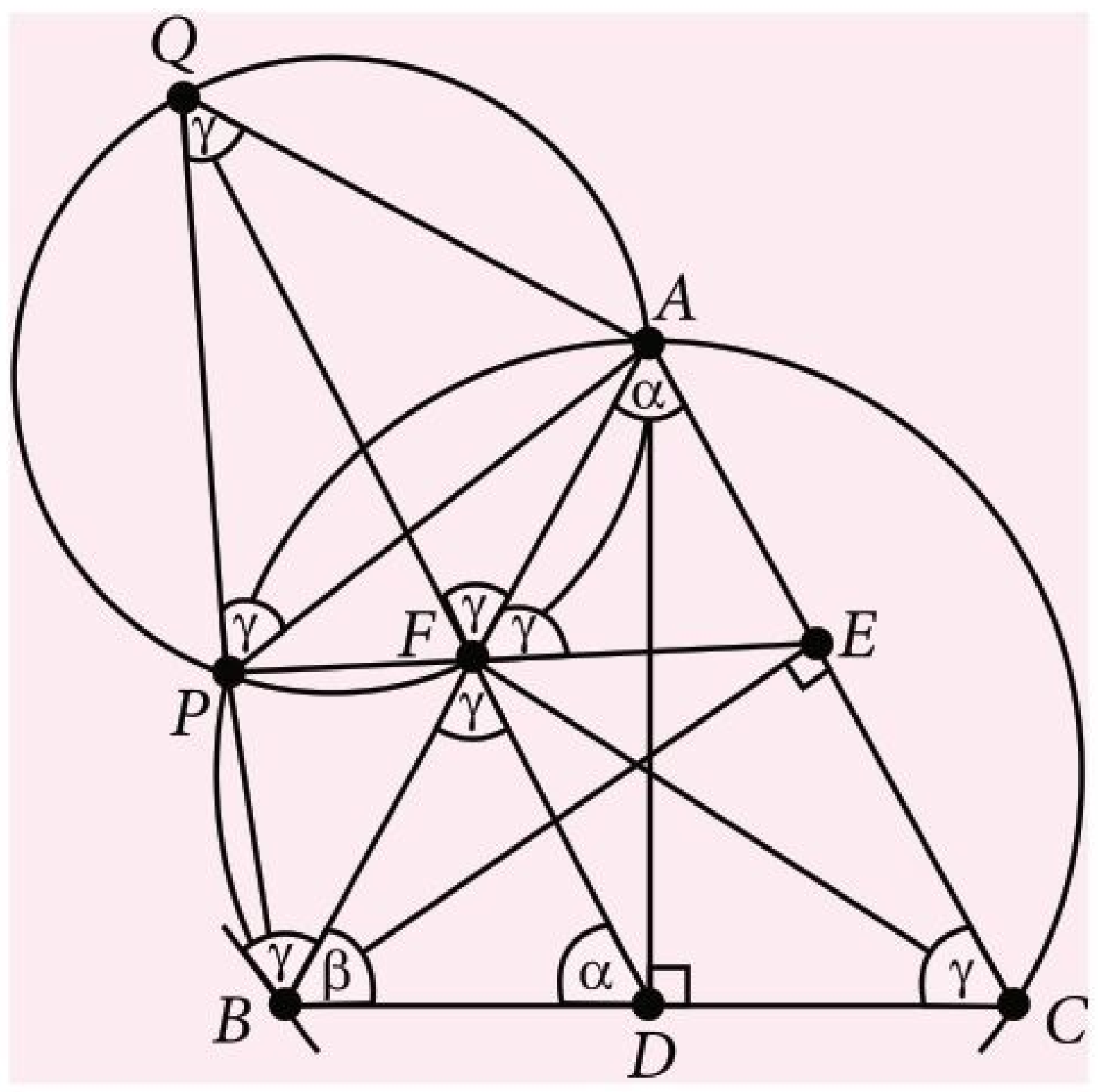
6. Integers can be written in bases other than the usual base 10. For example, the notation $(235)_7$ stands for the base 7 representation of the integer $2 \times 7^2 + 3 \times 7 + 5$ (which equals 124 in base 10). In general, if x, y and z are integers between 0 and $b - 1$, inclusive, then $(xyz)_b = xb^2 + yb + z$.

Find all triples (x, y, z) for which $(xyz)_{10} = 2(xyz)_7$, where each of x, y and z comes from the list 1, 2, 3, 4, 5, 6.

SOLUTIONS

1. The line EF intersects the circumcircle at two points. Depending on the choice of P, there are two different cases to consider.

Case 1: The point P lies on the ray EF (see Fig.).



Let $\angle CAB = \alpha$, $\angle ABC = \beta$ and $\angle BCA = \gamma$.

The quadrilaterals $BCEF$ and $CAFD$ are cyclic due to the right angles at D, E and F. So,

$$\angle BDF = 180° - \angle FDC = \angle CAF = \alpha,$$

$$\angle AFE = 180° - \angle EFB = \angle BCE = \gamma,$$

$$\angle DFB = 180° - \angle AFD = \angle DCA = \gamma.$$

Since P lies on the arc AB of the circumcircle, $\angle PBA < \angle BCA = \gamma$.

Hence, we have

$$\angle PBD + \angle BDF = \angle PBA + \angle ABD + \angle BDF < \gamma + \beta + \alpha = 180°,$$

and the point Q must lie on the extensions of BP and DF beyond the points P and F, respectively.

From the cyclic quadrilateral $APBC$, we get

$$\angle QPA = 180° - \angle APB = \angle BCA = \gamma = \angle DFB = \angle QFA.$$

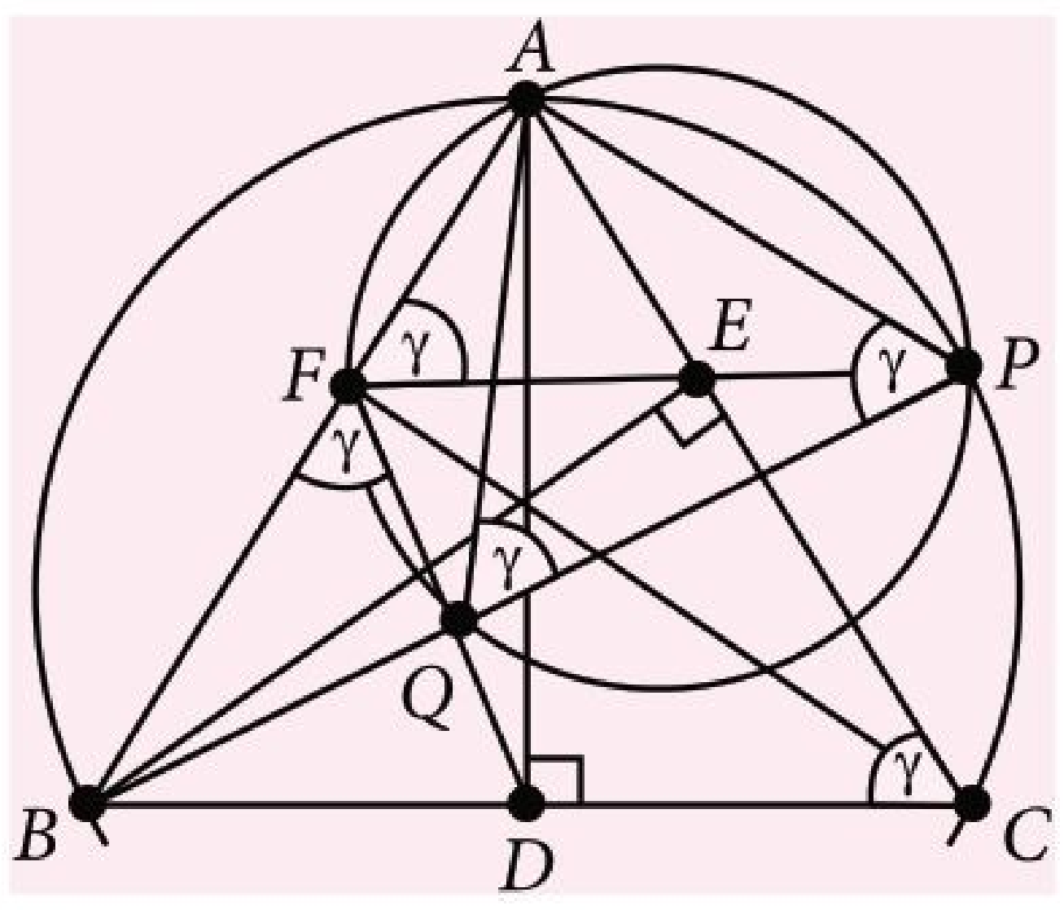
Hence, the quadrilateral $AQPF$ is cyclic.

Then $\angle AQP = 180° - \angle PFA = \angle AFE = \gamma$.

We have obtained that $\angle AQP = \angle QPA = \gamma$, so the triangle AQP is isosceles, $AP = AQ$.

Case 2: The point P lies on the ray FE (see Fig.). In this case the point Q lies inside the segment FD.

Similarly to the first case, we have



$\angle QPA = \angle BCA = \gamma = \angle DFB = 180° - \angle AFQ$.

Hence, the quadrilateral $AFQP$ is cyclic.

Then $\angle AQP = \angle AFP = \angle AFE = \gamma = \angle QPA$. The triangle AQP is isosceles again, $\angle AQP = \angle QPA$ and thus $AP = AQ$.

2. First, it is clear that all functions of the form $f(n) = n + c$ with a constant non-negative integer c satisfy the problem conditions since $(f(m) + n)(f(n) + m) = (n + m + c)^2$ is a square.

We are left to prove that there are no other functions. We start with the following Lemma. Suppose that $p \mid f(k) - f(l)$ for some prime p and positive integers k, l. Then $p \mid k - l$.

Proof : Suppose that $p^2 \mid f(k) - f(l)$, so $f(l) = f(k) + p^2a$ for some integer a. Take some positive integer $D > \max\{f(k), f(l)\}$ which is not divisible by p and set $n = pD - f(k)$. Then the positive numbers $n + f(k) = pD$ and $n + f(l) = pD + (f(l) - (f(k)) = p(D + pa)$ are both divisible by p but not by p^2. Now, applying the problem conditions, we get that both the numbers $(f(k) + n)(f(n) + k)$ and $(f(l) + n)(f(n) + l)$ are squares divisible by p (and thus by p^2); this means that the multipliers $f(n) + k$ and $f(n) + l$ are also divisible by p, therefore $p \mid (f(n) + k) - (f(n) + l) = k - l$ as well.

On the other hand, if $f(k) - f(l)$ is divisible by p but not by p^2, then choose the same number D and set $n = p^3D - f(k)$. Then the positive numbers $f(k) + n = p^3D$ and $f(l) + n = p^3D + (f(l) - f(k))$ are respectively divisible by p^3 (but not by p^4) and by p (but not by p^2). Hence in analogous way we obtain that the numbers $f(n) + k$ and $f(n) + l$ are divisible by p, therefore $p \mid (f(n) + k) - (f(n) + l) = k - l$.

We turn to the problem. First, suppose that $f(k) = f(l)$ for some $k, l \in \mathbb{N}$. Then by Lemma we have that $k - l$ is divisible by every prime number, so $k - l = 0$ or $k = l$. Therefore, the function f is injective.

Next, consider the numbers $f(k)$ and $f(k + 1)$. Since the number $(k + 1) - k = 1$ has no prime divisors, by Lemma the same holds for $f(k + 1) - f(k)$; thus $|f(k + 1) - f(k)| = 1$.

Now, let $f(2) - f(1) = q$, $|q| = 1$. Then we prove by induction that $f(n) = f(1) + q(n - 1)$. The base for $n = 1, 2$ holds by the definition of q. For the step, if $n > 1$ we have $f(n + 1) = f(n) \pm q = f(1) + q(n - 1) \pm q$. Since $f(n) \neq f(n - 2) = f(1) + q(n - 2)$, we get $f(n) = f(1) + qn$, are desired.

Finally, we have $f(n) = f(1) + q(n - 1)$. Then q cannot be -1 since otherwise for $n \geq f(1) + 1$ we have $f(n) \leq 0$ which is impossible. Hence $q = 1$ and $f(n) = (f(1) - 1) + n$ for each $n \in \mathbb{N}$, and $f(1) - 1 \geq 0$, as desired.

3. We start with some observations. First, from the definition of x_i it follows that for each positive integer k, we have

$$x_{4k-3} = x_{2k-1} = -x_{4k-2}$$
$$\text{and } x_{4k-1} = x_{4k} = -x_{2k} = x_k. \qquad \ldots\text{(i)}$$

Hence, denoting $S_n = \sum_{i=1}^{n} x_i$, we have

$$S_{4k} = \sum_{i=1}^{k}((x_{4k-3} + x_{4k-2}) + (x_{4k-1} + x_{4k}))$$

$$= \sum_{i=1}^{k}(0 + 2x_k) = 2S_k \qquad \ldots\text{(ii)}$$

$$S_{4k+2} = S_{4k} + (x_{4k+1} + x_{4k+2}) = S_{4k}. \qquad \ldots\text{(iii)}$$

Observe also that $S_n = \sum_{i=1}^{n} x_i \equiv \sum_{i=1}^{n} 1 = n \pmod 2$.

Now we prove by induction on k that $S_i \geq 0$ for all $i \leq 4k$. The base case is valid since $x_1 = x_3 = x_4 = 1$, $x_2 = -1$. For the induction step, assume that $S_i \geq 0$ for all $i \leq 4k$. Using the relations (i) – (iii), we obtain

$$S_{4k+4} = 2S_{k+1} \geq 0,\ S_{4k+2} = S_{4k} \geq 0,$$

$$S_{4k+3} = S_{4k+2} + x_{4k+3} = \frac{S_{4k+2} + S_{4k+4}}{2} \geq 0.$$

So, we are left to prove that $S_{4k+1} \geq 0$. If k is odd, then $S_{4k} = 2S_k \geq 0$; since k is odd, S_k is odd as well, so we have $S_{4k} \geq 2$ and hence $S_{4k+1} = S_{4k} + x_{4k+1} \geq 1$.

Conversely, if k is even, then we have $x_{4k+1} = x_{2k+1} = x_{k+1}$, hence $S_{4k+1} = S_{4k} + x_{4k+1} = 2S_k + x_{k+1} = S_k + S_{k+1} \geq 0$. The step is proved.

4. Suppose that for some n there exist the desired numbers; we may assume that $s_1 < s_2 < \ldots < s_n$. Surely $s_1 > 1$ since otherwise $1 - \frac{1}{s_1} = 0$. So we have $2 \leq s_1 \leq s_2 - 1 \leq \ldots \leq s_n - (n - 1)$, hence $s_i \geq i + 1$ for each $i = 1, 2, \ldots, n$. Therefore

$$\frac{51}{2010} = \left(1 - \frac{1}{s_1}\right)\left(1 - \frac{1}{s_2}\right)\ldots\left(1 - \frac{1}{s_n}\right)$$

$$\geq \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\ldots\left(1 - \frac{1}{n+1}\right)$$

$$= \frac{1}{2}\cdot\frac{2}{3}\ldots\frac{n}{n+1} = \frac{1}{n+1},$$

which implies

$$n+1 \geq \frac{2010}{51} = \frac{670}{17} > 39,$$

so $n \geq 39$.

Now we are left to show that $n = 39$ fits. Consider the set $\{2, 3, ..., 33, 35, 36, ..., 40, 67\}$ which contains exactly 39 numbers. We have

$$\frac{1}{2}\cdot\frac{2}{3}\cdots\frac{32}{33}\cdot\frac{34}{35}\cdots\frac{39}{40}\cdot\frac{66}{67} = \frac{1}{33}\cdot\frac{34}{40}\cdot\frac{66}{67}$$

$$= \frac{17}{670} = \frac{51}{2010} \qquad ...(i)$$

hence for $n = 39$ there exists a desired example.

5. Since we want the maximum number of people, we start with the largest of the given choices and see if we can make it work.

Is it possible that 80 people could have eaten both ice cream and cake?

If so, there would be at least 80 people who ate cake and at least $\frac{3}{2}(80) = 120$ people who ate ice cream. (Note that there can be overlap between these two sets of people.)

Is this possible?

It is if we say that exactly 120 people ate ice cream and exactly 80 people ate cake, with all of these 80 eating ice cream as well.

So an overlap of 80 is possible.

6. Since we are told that $(xyz)_b = xb^2 + yb + z$, then $(xyz)_{10} = 10^2x + 10y + z = 100x + 10y + z$ and $(xyz)_7 = 7^2x + 7y + z = 49x + 7y + z$.

From the given information

$$(xyz)_{10} = 2(xyz)_7$$

$$\Rightarrow \; 100x + 10y + z = 2(49x + 7y + z)$$

$$\Rightarrow \; 100x + 10y + z = 98x + 14y + 2z \Rightarrow 2x = 4y + z$$

Since the left side of this equation is an even integer $(2x)$, then the right side must also be an even integer. Since $4y$ is an even integer, then for $4y + z$ to be an even integer, it must be the case that z is an even integer.

This gives us three possibilities: $z = 2$, $z = 4$ and $z = 6$.

Case 1 : $z = 2$

Here, $2x = 4y + 2$ or $x = 2y + 1$.

We try the possible values for y :

$y = 1$ gives $x = 2(1) + 1 = 3$; this gives the triple $(x, y, z) = (3, 1, 2)$

$y = 2$ gives $x = 2(2) + 1 = 5$; this gives the triple $(x, y, z) = (5, 2, 2)$

If y is at least 3, then $x = 2y + 1$ is at least 7, which is impossible.

Therefore, there are two triples that work when $z = 2$.

Case 2: $z = 4$

Here, $2x = 4y + 4$ or $x = 2y + 2$.

We try the possible values for y :

$y = 1$ gives $x = 2(1) + 2 = 4$; this gives the triple $(x, y, z) = (4, 1, 4)$

$y = 2$ gives $x = 2(2) + 2 = 6$; this gives the triple $(x, y, z) = (6, 2, 4)$

If y is at least 3, then $x = 2y + 2$ is at least 8, which is impossible.

Therefore, there are two triples that work when $z = 4$.

Case 3: $z = 6$

Here, $2x = 4y + 6$ or $x = 2y + 3$.

We try the possible values for y :

$y = 1$ gives $x = 2(1) + 3 = 5$; this gives the triple $(x, y, z) = (5, 1, 6)$

If y is at least 2, then $x = 2y + 3$ is at least 7, which is impossible.

Therefore, there is one triple that works when $z = 6$.

Finally, the triples that satisfy the equation are $(x, y, z) = (3, 1, 2), (5, 2, 2), (4, 1, 4), (6, 2, 4), (5, 1, 6)$.

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SAMURAI SUDOKU

ANSWER - JULY 2021

6	9	1	2	4	8	5	7	3				4	6	1	5	8	9	2	3	7
7	8	5	3	9	1	4	2	6				9	3	2	7	6	4	8	5	1
4	2	3	6	7	5	8	9	1				5	7	8	1	2	3	4	6	9
9	5	7	4	1	6	2	3	8				7	9	4	6	5	1	3	2	8
1	4	2	8	3	9	6	5	7				1	2	6	9	3	8	7	4	5
8	3	6	5	2	7	9	1	4				8	5	3	2	4	7	9	1	6
5	1	4	9	6	3	7	8	2	3	1	9	6	4	5	8	7	2	1	9	3
3	6	9	7	8	2	1	4	5	7	6	2	3	8	9	4	1	5	6	7	2
2	7	8	1	5	4	3	6	9	4	5	8	2	1	7	3	9	6	5	8	4
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						6	5	4	8	3	7	1	9	2						
						8	2	7	1	9	6	5	3	4						
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3	4	8	6	7	9	1	2	5				7	1	2	5	9	4	3	8	6
6	7	2	5	1	8	3	4	9				3	6	9	1	7	8	4	2	5
7	1	6	4	8	2	9	5	3				2	3	5	8	4	7	9	6	1
8	9	4	1	5	3	7	6	2				6	9	7	3	5	1	8	4	2
5	2	3	9	6	7	8	1	4				1	8	4	9	6	2	5	7	3

YOU ASK
WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If $a < b < c < d$, then prove that for any real λ, the quadratic equation $(x - a)(x - c) + \lambda(x - b)(x - d) = 0$ has real roots. ***(Eklavya, Delhi)***

Ans. The given equation reduces to

$(1 + \lambda)x^2 - \{(a + c) + \lambda(b + d)\}x + (ac + \lambda bd) = 0$ ...(i)

Now, if the roots are real, then discriminant ≥ 0

$\Rightarrow \{a + c + \lambda(b + d)\}^2 - 4(1 + \lambda)(ac + \lambda bd) \geq 0$

$\Rightarrow \{(b + d)^2 - 4bd\}\lambda^2 + \{2(a + c)(b + d) - 4(ac + bd)\}\lambda + \{(a + c)^2 - 4ac\} \geq 0$

$\Rightarrow (b - d)^2\lambda^2 + 2\{(a + c)(b + d) - 2(ac + bd)\}\lambda + (a - c)^2 \geq 0$ ...(ii)

The discriminant of quadratic expression (ii) is

$D = 4[(a + c)(b + d) - 2(ac + bd)]^2 - 4(b - d)^2 (a - c)^2$

$= 4[ab + bc + ad + cd - 2ac - 2bd + (b - d)(a - c)]$
$\times [ab + bc + ad + cd - 2ac - 2bd - (b - d)(a - c)]$
$[\because x^2 - y^2 = (x + y)(x - y)]$

$= 4[2ab + 2cd - 2ac - 2bd][2bc + 2ad - 2ac - 2bd]$

$= 16[ab + cd - ac - bd][bc + ad - ac - bd]$

$= 16(a - b)(b - c)(c - d)(d - a) < 0$ $[\because a < b < c < d]$

Also, coeff. of $\lambda^2 = (b - d)^2$, which is +ve.

Hence, inequality (ii) is true for any real λ which proves that equation (i) has real roots for any real λ.

2. Solve the equation $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2+12x + 9)$ ***(Ankita, M.P.)***

Ans. The given equation reduces to

$$\frac{\log(6x^2+23x+21)}{\log(2x+3)}+\frac{\log(4x^2+12x+9)}{\log(3x+7)}=4$$

$$\Rightarrow \frac{\log(2x+3)(3x+7)}{\log(2x+3)}+2\frac{\log(2x+3)}{\log(3x+7)}=4$$

$$\Rightarrow 1+\frac{\log(3x+7)}{\log(2x+3)}+2\frac{\log(2x+3)}{\log(3x+7)}=4$$

$$\Rightarrow y+\frac{2}{y}=3 \qquad \left[\text{Putting } \frac{\log(3x+7)}{\log(2x+3)}=y\right]$$

$\Rightarrow y^2 - 3y + 2 = 0$ gives $y = 1, 2$

$\Rightarrow 3x + 7 = 2x + 3$ or $3x + 7 = (2x + 3)^2$

Solving $3x + 7 = 2x + 3 \Rightarrow x = -4$

Solving $3x + 7 = (2x + 3)^2 \Rightarrow 4x^2 + 9x + 2 = 0$

$$\Rightarrow x=\frac{-9\pm 7}{8}=-2,\frac{-1}{4}$$

Also, for $\dfrac{1}{\log(2x+3)}$ to be defined, $2x + 3 > 0$ and $2x + 3 \neq 1$

$\Rightarrow x>\dfrac{-3}{2}$ and $x \neq -1$ and for $\dfrac{1}{\log(3x+7)}$ to be defined, $3x + 7 > 0$ and $3x + 7 \neq 1$

$\Rightarrow x>\dfrac{-7}{3}$ and $x \neq -2$

Hence, the only value of x satisfying the above conditions is $\dfrac{-1}{4}$.

3. Find the range of $f(x) = \dfrac{x}{1-x^2}$.

(Harman Singh Kalsi)

Ans. Let $y = \dfrac{x}{1-x^2}$

$\Rightarrow y(1 - x^2) = x \Rightarrow x^2y + x - y = 0$

$$\therefore \quad x=\frac{-1\pm\sqrt{1+4y^2}}{2}$$

For x to be defined, $1 + 4y^2 \geq 0$, which is true for all $y \in R$.

Also, $x \neq \pm 1$, for any value of $y \in R$

$\therefore$ Range of $f(x)$ is R.

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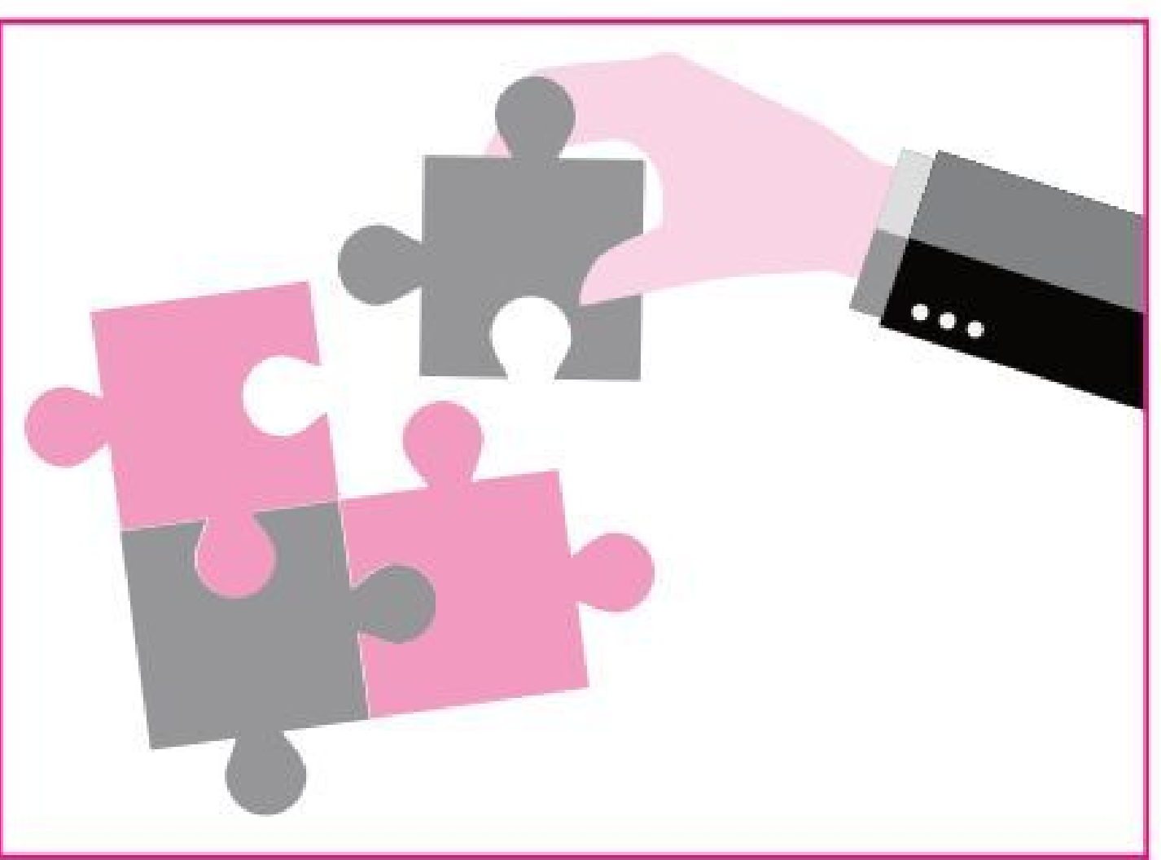
PUZZLE
CORNER
ANSWER · JULY 2021



13+ 6	2	5	11+ 4	2– 1	60× 3
11+ 1	6	2	5	3	4
4	24× 3	6= 6	6× 1	2	5
2	4	10+ 1	3	30× 5	6
45× 3	5	4	4– 2	2– 6	1
5	1	3	6	4	2= 2

JEE WORK OUTS

1. The value of $\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}+\sin\frac{8\pi}{7}$ is

(a) 1 (b) $\frac{\sqrt{7}}{2}$ (c) $\frac{3\sqrt{3}}{4}$ (d) $\frac{\sqrt{15}}{4}$

2. $\lim_{x\to 0}\left(\frac{10\sin 9x}{9\sin 10x}\right)\left(\frac{8\sin 7x}{7\sin 8x}\right)\left(\frac{6\sin 5x}{5\sin 6x}\right)\left(\frac{4\sin 3x}{3\sin 4x}\right)\left(\frac{\sin x}{\sin 2x}\right)=$

(a) $\frac{63}{256}$ (b) $\frac{1}{6}$ (c) $\frac{6}{5}$ (d) $\frac{1}{2}$

3. The domain of the function

$f(x)=\sqrt{7-3x}+\log_e x$ is

(a) $0 < x < \infty$ (b) $\frac{7}{3}\le x<\infty$

(c) $0<x\le\frac{7}{3}$ (d) $-\infty < x < 0$

4. The range of the function $f(x)=\frac{1}{2-\cos 3x}$ is

(a) $(-2, \infty)$ (b) $[-2, 3]$

(c) $\left(\frac{1}{3},2\right)$ (d) $\left[\frac{1}{3},1\right]$

5. If $f(x) = \log_{x2}(\log x)$, then $f'(x)$ at $x = e$ is

(a) 0 (b) 1 (c) $\frac{1}{e}$ (d) $\frac{1}{2e}$

6. The function $f(x) = [x]$, where $[x]$ denotes greatest integer function is continuous at

(a) -2 (b) 1.5 (c) 4 (d) 1

7. The function $f(x) = a\sin|x| + be^{|x|}$ is differentiable at $x = 0$ when

(a) $3a + b = 0$ (b) $3a - b = 0$

(c) $a + b = 0$ (d) $a - b = 0$

8. Let $f(x)$ be a differentiable function and $f'(4) = 5$. Then $\lim_{x\to 2}\frac{f(4)-f(x^2)}{x-2}$ equals

(a) 0 (b) 5

(c) 20 (d) -20

9. The value of $\frac{d}{dx}[x(x-1)(x-2)(x-3)]$ at $x=3$ is

(a) 0 (b) 3

(c) 6 (d) 24

10. $\lim_{x\to 0} x\sin(e^{1/x})$

(a) is equal to 0 (b) is equal to 1

(c) is equal to $\frac{e}{2}$ (d) does not exist

11. If $f(x)=\sin^{-1}\left(\frac{1-\cos 2x}{2\sin x}\right)$, then $|f'(x)|$ is equal to

(a) $|\sin x|$ (b) x

(c) 0 (d) 1

12. If $A=\left\{x:\frac{\pi}{6}\le x\le\frac{\pi}{3}\right\}$ and $f(x) = \cos x - x(1 + x)$, then $f(A)$ is equal to

(a) $\left[\frac{1}{2}-\frac{\pi}{3}-\frac{\pi^2}{9},\frac{\sqrt{3}}{2}-\frac{\pi}{6}-\frac{\pi^2}{36}\right]$

(b) $\left[\frac{1}{2}+\frac{\pi}{3}-\frac{\pi^2}{9},\frac{\sqrt{3}}{2}+\frac{\pi}{6}-\frac{\pi^2}{36}\right]$

(c) $\left(\frac{1}{2}-\frac{\pi}{3}-\frac{\pi^2}{9},\frac{\sqrt{3}}{2}-\frac{\pi}{6}-\frac{\pi^2}{36}\right)$

(d) None of these

13. If the function $f(x)=\left[\dfrac{(x-2)^3}{a}\right]$ sin $(x - 2) + a$ cos $(x - 2)$, (where $[\cdot]$ denotes the greatest integer function) is continuous and differentiable in (4, 6), then

(a) $a \in [8, 64]$ (b) $a \in (0, 8]$
(c) $a \in [64, \infty)$ (d) None of these

14. If $f(x)=\begin{cases} 1/|x|; & |x|\geq 1 \\ ax^2+b; & -1<x<1 \end{cases}$ is differentiable $\forall\, x$, then value of a and b is

(a) $a = 1/2$, $b = -3/2$ (b) $a = -1/2$, $b = 3/2$
(c) $a = 3/2$, $b = 1/2$ (d) None of these

15. If $f''(x) > 0$ and $f'(1) = 0$ such that $g(x) = f(\cot^2 x + 2\cot x + 2)$, where $0 < x < \pi$ then the interval in which $g(x)$ is decreasing is

(a) $(0, \pi)$ (b) $\left(\dfrac{\pi}{2}, \pi\right)$

(c) $\left(\dfrac{3\pi}{4}, \pi\right)$ (d) $\left(0, \dfrac{3\pi}{4}\right)$

16. If $I=\displaystyle\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx$, then I equals

(a) $\log|x|+\log\left|1+\sqrt{1-x^2}\right|+\sin^{-1}\sqrt{x}+C$

(b) $\log|x|-\log\left|1-\sqrt{1-x^2}\right|+\tan^{-1}x+C$

(c) $\log|x|-\log\left|1+\sqrt{1-x^2}\right|-\sin^{-1}x+C$

(d) $\log\left|1+\sqrt{1-x^2}\right|-\log|x|+\cos^{-1}x+C$

17. If $f(x)=\tan^{-1}x+\ln\sqrt{1+x}-\ln\sqrt{1-x}$. The integral of $(1/2)\,f'(x)$ with respect to x^4 is

(a) $e^{-x^4}+c$ (b) $-\ln(1 - x^4) + c$

(c) $e^{\sqrt{1-x^4}}+c$ (d) $\ln(1 + x^2) + c$

18. The value of $\displaystyle\int_1^{16}\tan^{-1}\sqrt{\sqrt{x}-1}\,dx$ is

(a) $\dfrac{16\pi}{3}+2\sqrt{3}$ (b) $\dfrac{4}{3}\pi-2\sqrt{3}$

(c) $\dfrac{4}{3}\pi+2\sqrt{3}$ (d) $\dfrac{16}{3}\pi-2\sqrt{3}$

19. If f is a continuous function such that $\displaystyle\int_0^x f(t)dt \to \infty$ as $|x| \to \infty$, then for all $k \in R$, then $k^2x^2+\displaystyle\int_0^x f(t)dt-a=0\,(a>0)$ has

(a) all roots in $(-\infty, 0)$
(b) all roots in $(0, \infty)$
(c) odd number of roots in $(-\infty, 0)$ and odd number of roots in $(0, \infty)$
(d) None of these

20. Through any point (x, y) of a curve which passes through the origin, lines are drawn parallel to the coordinate axes. The curve, given that it divides the rectangle formed by the two lines and the axes into two areas, one of which is twice the other, represents a family of

(a) circles (b) parabolas
(c) hyperbolas (d) straight lines

21. The area bounded by the curve $y = x^4 - 2x^3 + x^2 + 3$, the axis of abscissa and two ordinates corresponding to the points of minimum of the function $y(x)$ (in sq. units) is

(a) 10/3 (b) 27/10
(c) 21/10 (d) None of these

22. Solution of the equation $y=x\dfrac{dy}{dx}+\dfrac{dx}{dy}$ represents

(a) Family of straight lines and a parabola
(b) Family of straight lines and a hyperbola
(c) Family of circles and parabola
(d) None of these

23. The differential equation of family of parabola with foci at the origin and axis along the x axes is

(a) $y\left(\dfrac{dy}{dx}\right)^2+2x\dfrac{dy}{dx}-y=0$

(b) $x\left(\dfrac{dy}{dx}\right)^2+2y\dfrac{dy}{dx}-y=0$

(c) $y\left(\dfrac{dy}{dx}\right)^2+2x\dfrac{dy}{dx}+y=0$

(d) None of these

24. If z_1, z_2 are two complex numbers such that $\left|\frac{z_1-z_2}{z_1+z_2}\right|=1$ and $iz_1 = Kz_2$, where $K \in R$, then the angle between $z_1 - z_2$ and $z_1 + z_2$ is

(a) $\tan^{-1}\left(\frac{2K}{K^2+1}\right)$ (b) $\tan^{-1}\left(\frac{K}{1-K^2}\right)$

(c) $-2\tan^{-1}K$ (d) $2\tan^{-1}K$

25. If α is a complex n^{th} root of unity and if Z_1 and Z_2 are two complex numbers, then $\sum_{r=0}^{n-1}|Z_1+\alpha^r Z_2|^2 =$

(a) $n^2|Z_1 + Z_2|^2$ (b) $\left(\frac{Z_1}{n}+\frac{Z_2}{n}\right)^2$

(c) $n(|Z_1|^2 + |Z_2|^2)$ (d) $n^2(|Z_1|^2 + |Z_2|^2)$

26. Two persons A and B throw a die alternately till one of them get a "six" and wins the game. The probability of winning of B if A starts first is

(a) $\frac{6}{11}$ (b) $\frac{5}{11}$

(c) $\frac{3}{11}$ (d) None of these

27. A person throws a dice while he gets a number greater than 2. The probability that he gets a 6 in the last throw is

(a) 2/3 (b) 1/4 (c) 1/3 (d) 1/12

28. The values of λ for which the system of equations $x + y - 3 = 0$, $(1 + \lambda)x + (2 + \lambda)y - 8 = 0$, $x - (1 + \lambda)y + (2 + \lambda) = 0$ is consistent are

(a) $-5/3, 1$ (b) $2/3, -3$

(c) $-1/3, -3$ (d) $0, 0$

29. The number of values of k for which the system of equations $(k + 1)x + 8y = 4k$, $kx + (k + 3)y = 3k - 1$ has no solution is

(a) 0 (b) 1 (c) 2 (d) infinite

30. If $A=\begin{bmatrix}0 & 5\\ 0 & 0\end{bmatrix}$ and $f(x) = 1 + x + x^2 + ... + x^{16}$, then $f(A)$ is equal to

(a) 0 (b) $\begin{bmatrix}1 & 5\\ 0 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 5\\ 0 & 0\end{bmatrix}$ (d) $\begin{bmatrix}0 & 5\\ 1 & 1\end{bmatrix}$

31. If $|\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=5$ and $\vec{a}\perp(\vec{b}+\vec{c}), \vec{b}\perp(\vec{c}+\vec{a}), \vec{c}\perp(\vec{a}+\vec{b})$ then $|\vec{a}+\vec{b}+\vec{c}|$ is equal to

(a) 50 (b) 25

(c) $5\sqrt{2}$ (d) 12

32. If lines $\frac{x-1}{2}=\frac{y-2}{x_1}=\frac{z-3}{x_2}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ lies in the same plane, then for equation $x_1t^2 + (x_2 + 2)t + a = 0$, which of the options holds good?

(a) $2x_1 - x_2 = 1$

(b) Sum of roots of given equation = -2

(c) $2x_1 + x_2 = -4$

(d) Sum of roots of given equation = 0

33. The variance of the first n natural numbers is

(a) $\frac{n^2-1}{12}$ (b) $\frac{n^2-1}{6}$

(c) $\frac{n^2+1}{6}$ (d) $\frac{n^2+1}{12}$

SOLUTIONS

1. (b): Let $S=\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}+\sin\frac{8\pi}{7}$ and

$C=\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{8\pi}{7}$

Now, $C + iS = \alpha + \alpha^2 + \alpha^4$ ...(i)

where, $\alpha=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$, is complex 7th root of unity.

Also, $C - iS = \alpha^6 + \alpha^5 + \alpha^3$ ...(ii)

$\therefore\ \alpha^6=\bar{\alpha}, \alpha^5=\bar{\alpha}^2, \alpha^3=\bar{\alpha}^4$

Adding (i) and (ii), we get

$$2C=\alpha+\alpha^2+\alpha^3+...+\alpha^6=\frac{\alpha^7-\alpha}{\alpha-1}=-1 \quad (\because\ \alpha^7=1)$$

$\Rightarrow C = -1/2$

Multiplying (i) and (ii), we get

$C^2+S^2=2 \Rightarrow S=\frac{\sqrt{7}}{2}$

2. (d): $\lim_{x\to0}\left(\frac{10\sin 9x}{9\sin 10x}\right)\left(\frac{8\sin 7x}{7\sin 8x}\right)\left(\frac{6\sin 5x}{5\sin 6x}\right)\left(\frac{4\sin 3x}{3\sin 4x}\right)\left(\frac{\sin x}{\sin 2x}\right)$

$$=\lim_{x\to0}\frac{\frac{\sin 9x}{9x}}{\frac{\sin 10x}{10x}}\cdot\frac{\frac{\sin 7x}{7x}}{\frac{\sin 8x}{8x}}\cdot\frac{\frac{\sin 5x}{5x}}{\frac{\sin 6x}{6x}}\cdot\frac{\frac{\sin 3x}{3x}}{\frac{\sin 4x}{4x}}\cdot\frac{\frac{\sin x}{x}}{2\times\frac{\sin 2x}{2x}}$$

$$=1\cdot1\cdot1\cdot1\cdot\frac{1}{2\times1}=\frac{1}{2} \quad \left[\because\ \lim_{x\to0}\frac{\sin x}{x}=1\right]$$

3. (c) : We have, $f(x)=\sqrt{7-3x}+\log_e x$

Let $g(x)=\sqrt{7-3x}$ and $h(x)=\log_e x$

Now $g(x)$ will be defined, when $7 - 3x \geq 0$

i.e., $x \leq \frac{7}{3}$ $\therefore$ domain of g is $-\infty < x \leq \frac{7}{3}$

Also $h(x)$ will be defined when $x > 0$

$\therefore$ domain of h is $0 < x < \infty$

So, domain of $f(x)$ is domain of $g \cap$ domain of h

i.e. $0 < x \leq \frac{7}{3}$.

4. (d) : We have, $f(x)=\dfrac{1}{2-\cos 3x}$

We know that, $-1 \leq \cos 3x \leq 1$

$\therefore$ $f(x)$ will be minimum when $\cos 3x = -1$ and minimum value of $f(x)$ is $\frac{1}{3}$.

Again $f(x)$ will be maximum when $\cos 3x = 1$ and maximum value of $f(x)$ is 1.

Therefore, the required range is $\frac{1}{3} \leq f(x) \leq 1$ *i.e.* $\left[\frac{1}{3}, 1\right]$

5. (d): $f(x)=\log_{x^2}(\log x)=\dfrac{\log_e(\log_e x)}{\log_e x^2}$

$$\Rightarrow \ f(x)=\frac{\log_e(\log_e x)}{2\log_e x}$$

$$\therefore \ f'(x)=\frac{1}{2}\cdot\frac{\log_e x\cdot\dfrac{1}{x\log_e x}-\dfrac{1}{x}\log_e(\log_e x)}{(\log_e x)^2}$$

Now $f'(x)$ at $x=e=\dfrac{1}{2}\dfrac{\left(\dfrac{1}{e}-\dfrac{1}{e}\times 0\right)}{(1)^2}=\dfrac{1}{2e}$

6. (b) : $f(x) = [x]$ is continuous at $x = 1.5$

$\because \lim_{x\to 1.5^-}[x]=1$ and $\lim_{x\to 1.5^+}[x]=1$

7. (c) : $f(x) = a\sin|x| + be^{|x|}$

$$\therefore \ f(x)=\begin{cases} a\sin x+be^{x}, & \text{when } x\geq 0\\ -a\sin x+be^{-x}, & \text{when } x<0\end{cases}$$

$$f'(x)=\begin{cases} a\cos x+be^{x}, & \text{when } x\geq 0\\ -a\cos x-be^{-x}, & \text{when } x<0\end{cases}$$

Since, at $x = 0$, the function is differentiable.

$\therefore$ $Rf'(0) = Lf'(0)$

$\Rightarrow$ $a + b = -a - b$ $\Rightarrow$ $a + b = 0$

8. (d) : $\displaystyle\lim_{x\to 2}\frac{f(4)-f(x^2)}{x-2}=\lim_{x\to 2}\frac{f(4)-f(x^2)}{x^2-4}\cdot(x+2)$

$$=-\lim_{x\to 2}\frac{f(x^2)-f(4)}{x^2-4}\cdot(x+2)$$

$$=-\lim_{h\to 0}\frac{f(h+4)-f(4)}{h}\times\lim_{x\to 2}(x+2)$$

$$\begin{bmatrix}\text{Let } x^2-4=h \ \therefore x^2=h+4\\ x\to 2 \Rightarrow h\to 0\end{bmatrix}$$

$= -4f'(4) = -4 \times 5 = -20$

9. (c) : $\dfrac{d}{dx}[x(x-1)(x-2)(x-3)]$

$=\dfrac{d}{dx}[(x^2-3x)(x^2-3x+2)]$

$= (x^2 - 3x)\cdot(2x - 3) + (x^2 - 3x + 2)(2x - 3)$

$= (2x - 3)(x^2 - 3x + x^2 - 3x + 2)$

$= (2x - 3)(2x^2 - 6x + 2)$

$= 2(2x - 3)(x^2 - 3x + 1)$

$\therefore$ At $x = 3$, the value of $\dfrac{d}{dx}[x(x-1)(x-2)(x-3)]$

$= 2 \times (6 - 3)(9 - 9 + 1)$

$= 2 \times 3 = 6$

10. (a) : We have, $-1 \leq \sin x \leq 1$

$\therefore$ $-1 \leq \sin e^{1/x} \leq 1$

$\Rightarrow$ $-x \leq x\sin e^{1/x} \leq x$

Since, $\lim_{x\to 0} -x = \lim_{x\to 0} x = 0$

$\therefore$ By Sandwich Theorem, we get

$$\lim_{x\to 0} x\sin e^{1/x}=0$$

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11. (d) : $f(x)=\sin^{-1}\left(\frac{1-\cos 2x}{2\sin x}\right)$

$$=\sin^{-1}\left(\frac{2\sin^2 x}{2\sin x}\right)=\sin^{-1}(\sin x)$$

$\Rightarrow f(x) = x \Rightarrow f'(x) = 1$

So, $|f'(x)| = 1$

12. (a) : We have $f(x) = \cos x - x(1 + x)$

Since, in the interval $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$, $\cos x$ decreases and $x(1 + x)$ increases.

$\therefore\ f(x)$ decreases in $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$.

$$\therefore\ f\left(\frac{\pi}{3}\right)\le f(x)\le f\left(\frac{\pi}{6}\right);\ x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$$

Hence, $f(A)=\left[\frac{1}{2}-\frac{\pi}{3}\left(1+\frac{\pi}{3}\right),\frac{\sqrt{3}}{2}-\frac{\pi}{6}\left(1+\frac{\pi}{6}\right)\right]$

$$=\left[\frac{1}{2}-\frac{\pi}{3}-\frac{\pi^2}{9},\frac{\sqrt{3}}{2}-\frac{\pi}{6}-\frac{\pi^2}{36}\right]$$

13. (c) : We have, $x \in (4, 6) \Rightarrow 2 < x - 2 < 4$

$$\Rightarrow\ \frac{8}{a}<\frac{(x-2)^3}{a}<\frac{64}{a},\ a>0$$

For $f(x)$ to be continuous and differentiable in $(4, 6)$, $\left[\frac{(x-2)^3}{a}\right]$ must attain a constant value for $x \in (4, 6)$.

Clearly, this is possible only when $a \ge 64$
In that case, we have
$f(x) = a\cos(x - 2)$ which is continuous and differentiable.
$\therefore\ a \in [64, \infty)$

14. (b): $f(x)=\begin{cases}1/|x|; & |x|\ge 1\\ ax^2+b; & -1<x<1\end{cases}$

$$f(x)=\begin{cases}-\frac{1}{x}; & x\le -1\\ ax^2+b; & -1<x<1\\ \frac{1}{x}; & x\ge 1\end{cases}$$

Since $f(x)$ is continuous at $x = 1$.
$\therefore\ a\cdot 1 + b = 1$ ...(i)
Also, $f(x)$ is differentiable at $x = 1$.

$$\therefore\ 2ax=-\frac{1}{x^2}$$

At $x = 1$, $2a = -1 \Rightarrow a = -1/2$
Using (i), we get, $b = 3/2$

15. (d): Here, $g(x) = f(\cot^2 x + 2\cot x + 2)$
$\Rightarrow g'(x) = f'(\cot^2 x + 2\cot x + 2)$
$\{-2\cot x\operatorname{cosec}^2 x - 2\operatorname{cosec}^2 x\}$
For $g(x)$ to be decreasing, $g'(x) < 0$
$\Rightarrow f'\{(\cot x + 1)^2 + 1\}\cdot(-2\operatorname{cosec}^2 x)(\cot x + 1) < 0$
$\Rightarrow f'\{(\cot x + 1)^2 + 1\}\cdot(\cot x + 1) > 0$ ...(i)
{as $f''(x) > 0 \Rightarrow f'(x)$ is increasing, then

$$f'\left\{(\cot x+1)^2+1>f'(1)=0\ \forall x\in\left(0,\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{4},\pi\right)\right\}$$

Thus, equation (i) holds, if $\cot x + 1 > 0$

$$\Rightarrow\ \cot x>-1\ \forall x\in\left(0,\frac{3\pi}{4}\right)$$

16. (c) : $I=\int\left(\frac{1}{x}\right)\frac{1-x}{\sqrt{1-x^2}}dx=\int\frac{dx}{x\sqrt{1-x^2}}-\int\frac{dx}{\sqrt{1-x^2}}$

In the first integral, put $x=\frac{1}{t}$, we get

$$I=\int\frac{(-1/t^2)\,dt}{\frac{1}{t}\sqrt{1-\frac{1}{t^2}}}-\sin^{-1}x=-\int\frac{dt}{\sqrt{t^2-1}}-\sin^{-1}x$$

$$=-\log|t+\sqrt{t^2-1}|-\sin^{-1}x+C$$

$$=-\log\left|\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right|-\sin^{-1}x+C$$

$$=\log|x|-\log|1+\sqrt{1-x^2}|-\sin^{-1}x+C$$

17. (b): $f(x)=\tan^{-1}x+\ln\sqrt{1+x}-\ln\sqrt{1-x}$

$$f'(x)=\frac{1}{1+x^2}+\frac{1}{2(1+x)}+\frac{1}{2(1-x)}$$

$$=\frac{1}{1+x^2}+\frac{1}{1-x^2}=\frac{2}{(1-x^4)}$$

$$\Rightarrow\ \frac{1}{2}f'(x)=\frac{1}{1-x^4}\ \Rightarrow\ \int\frac{1}{2}f'(x)dx^4=\int\frac{dx^4}{1-x^4}$$

$= -\ln(1 - x^4) + c$

18. (d): Integrating by parts, the given integral is equal to

$$\left[x\tan^{-1}\sqrt{\sqrt{x}-1}\right]_1^{16}-\int_1^{16}\frac{x}{\sqrt{x}}\frac{1}{4\sqrt{x}\sqrt{\sqrt{x}-1}}dx$$

$$=\frac{16}{3}\pi-\frac{1}{4}\int_1^{16}\frac{dx}{\sqrt{\sqrt{x}-1}}$$

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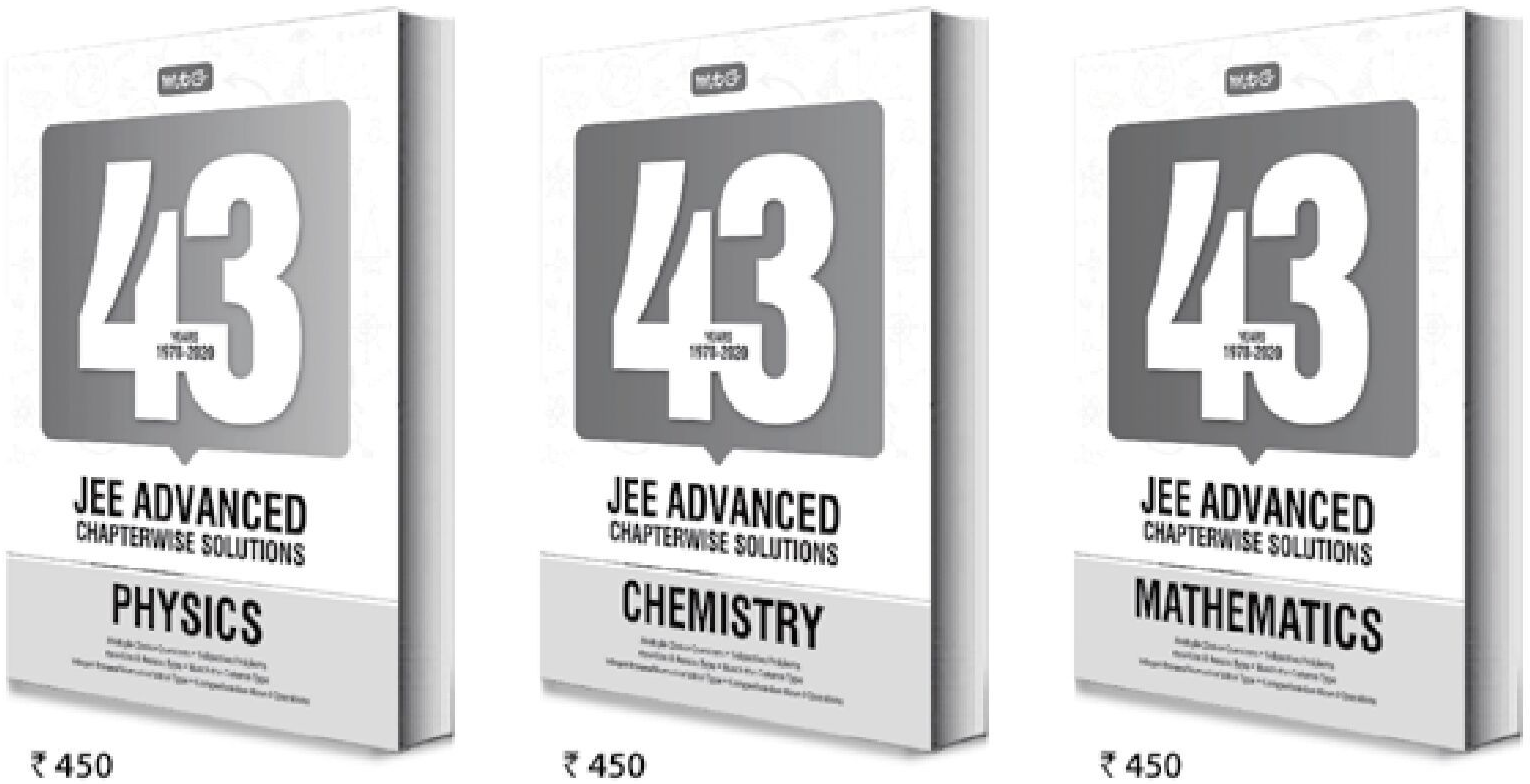

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$$=\frac{16}{3}\pi-\frac{1}{4}\int_0^{\sqrt{3}}\frac{4t(1+t^2)}{t}dt \qquad (\text{Put } \sqrt{x}=1+t^2)$$

$$=\frac{16}{3}\pi-\left(\sqrt{3}+\sqrt{3}\right)=\frac{16}{3}\pi-2\sqrt{3}$$

19. (c) : Let $g(x)=k^2x^2+\int_0^x f(t)dt-a$ ...(i)

Since, $a>0$ and $\int_0^x f(t)dt\to\infty$ as $x\to\pm\infty$ (given)

$$\Rightarrow \quad g(0)=0+\int_0^0 f(t)dt-a=-a<0$$

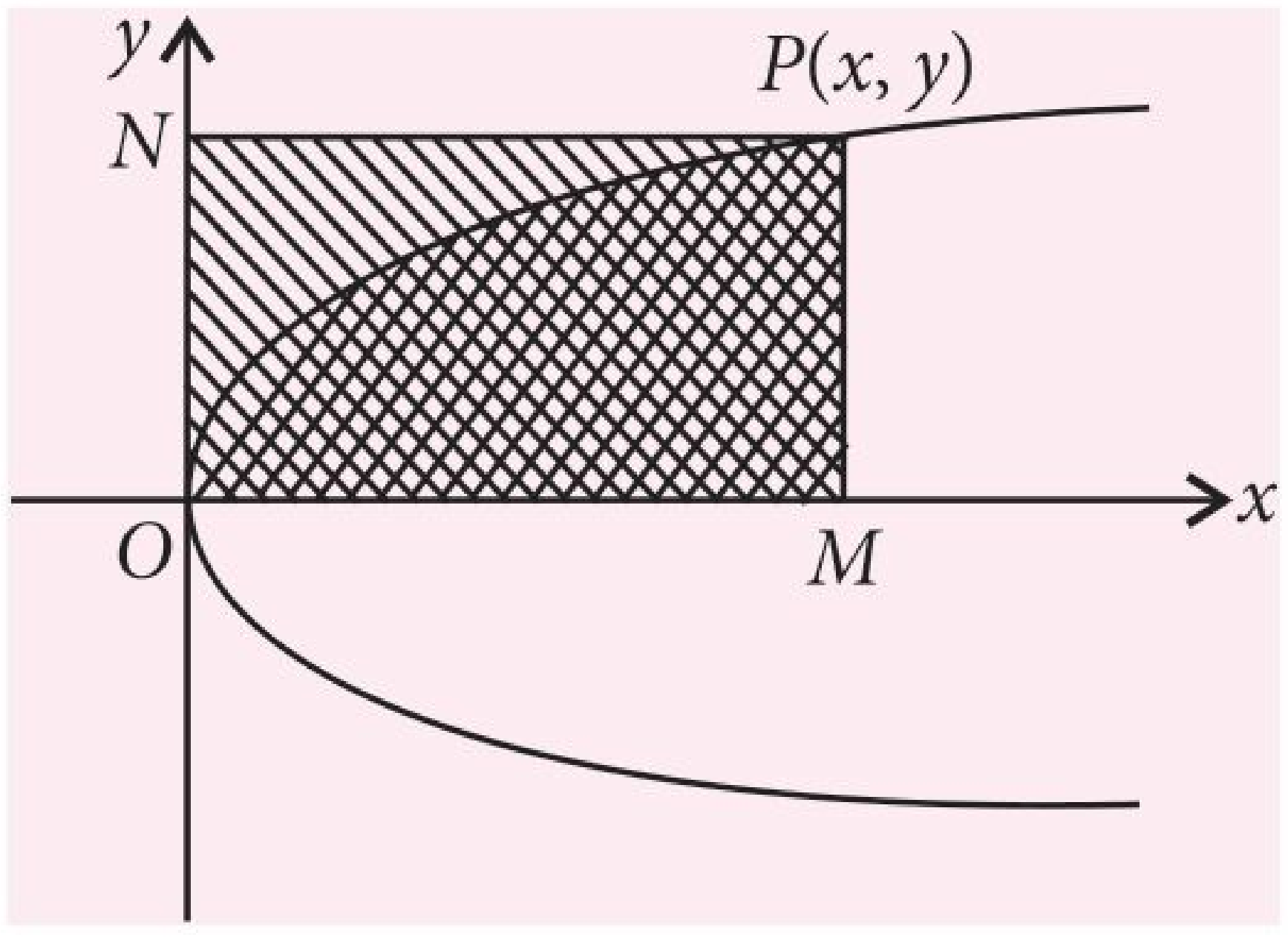
and $g(\infty)=\infty>0$

Also, $g(-\infty)=\infty>0$

$\Rightarrow$ $g(x)$ is continuous in $(-\infty, \infty)$.

Hence, $g(x)=0$ has odd number of roots in $(-\infty, 0)$ and odd number of roots in $(0, \infty)$.

20. (b): Let $P(x, y)$ be the point on the curve passing through the origin $O(0, 0)$, and let PN and PM be the lines parallel to the x-axes and y-axes, respectively. If the equation of the curve is $y = y(x)$, the area POM equals $\int_0^x y\,dx$ and the area PON equals $xy-\int_0^x ydx$



Assuming that $2(POM) = PON$, we have

$$2\int_0^x y\,dx=xy-\int_0^x y\,dx \Rightarrow 3\int_0^x y\,dx=xy.$$

Differentiating both sides, we get

$$3y=x\frac{dy}{dx}+y \Rightarrow 2y=x\frac{dy}{dx} \Rightarrow \frac{dy}{y}=2\frac{dx}{x}$$

On integrating both sides, we get

$\Rightarrow$ $\log|y| = 2\log|x| + \log C \Rightarrow y = Cx^2$, with C being a constant.

This solutions represents a parabola. We will get a similar result if we had started instead with $2(PON) = POM$.

21. (d): Given curve is

$y = x^4 - 2x^3 + x^2 + 3$

$$\therefore \quad \frac{dy}{dx}=4x^3-6x^2+2x$$

$$\frac{d^2y}{dx^2}=12x^2-12x+2$$

For maxima and minima, put $\frac{dy}{dx}=0$

$\therefore$ $4x^3 - 6x^2 + 2x = 0$

$$\Rightarrow \quad x=0,\frac{1}{2},1$$

$$\therefore \quad \left[\frac{d^2y}{dx^2}\right]_{x=0}=2,\left[\frac{d^2y}{dx^2}\right]_{x=\frac{1}{2}}=-1 \text{ and } \left[\frac{d^2y}{dx^2}\right]_{x=1}=2$$

$\therefore$ Points of minima are $x = 0$ and $x = 1$

$$\therefore \text{ Required area}=\int_0^1(x^4-2x^3+x^2+3)\,dx$$

$$=\left[\frac{x^5}{5}-\frac{2x^4}{4}+\frac{x^3}{3}+3x\right]_0^1$$

$$=\frac{1}{5}-\frac{2}{4}+\frac{1}{3}+3=\frac{1}{5}-\frac{1}{2}+\frac{1}{3}+3=\frac{91}{30}\text{ sq.units}$$

22. (a): Putting $\frac{dy}{dx}=P$ in given equation, we get

$$y=xP+\frac{1}{P}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=P+x\frac{dP}{dx}-\frac{1}{P^2}\frac{dP}{dx}$$

$$\Rightarrow \quad P=P+x\frac{dP}{dx}-\frac{1}{P^2}\frac{dP}{dx} \qquad \left\{\because \frac{dy}{dx}=P\right\}$$

$$\Rightarrow \quad \frac{dP}{dx}=0 \quad\text{or}\quad P^2=\frac{1}{x} \quad \Rightarrow P=C \quad\text{or}\quad \left(\frac{dy}{dx}\right)^2=\frac{1}{x}$$

Put these value in given equation, we get

$y=Cx+\frac{1}{C}$, which is equation of family of straight lines.

$$\text{and}\quad y^2=\left(Px+\frac{1}{P}\right)^2=P^2x^2+2x+\frac{1}{P^2}$$

Put $P^2=\frac{1}{x}$, we get

$y^2 = x + 2x + x = 4x$,

which represents a parabola.

23. (a) : Here,

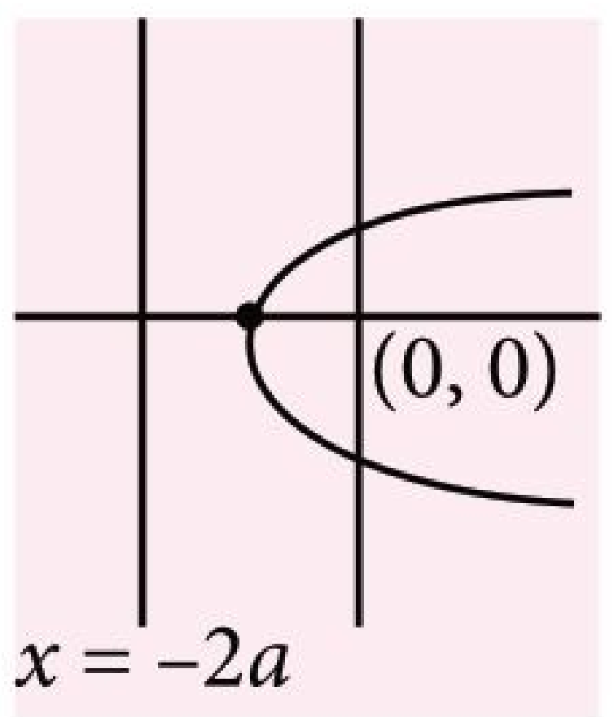
Distance from focus = distance from directrix

$\Rightarrow$ $x^2 + y^2 = (2a + x)^2$

$\Rightarrow$ $y^2 = 4a(a + x)$ ...(i)

$$\Rightarrow \quad 2y\frac{dy}{dx} = 4a(0+1)$$

$$\Rightarrow \quad a = \frac{y}{2}\frac{dy}{dx}$$



Using (i), $y^2 = 2y\dfrac{dy}{dx}\left(\dfrac{y}{2}\dfrac{dy}{dx} + x\right)$

$$\Rightarrow \quad y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$$

24. (d): $\dfrac{z_1 - z_2}{z_1 + z_2} = \cos\alpha + i\sin\alpha$

$$\Rightarrow \quad \frac{2z_1}{-2z_2} = \frac{\cos\alpha + i\sin\alpha + 1}{\cos\alpha + i\sin\alpha - 1}$$

$$= \frac{2\cos^2(\alpha/2) + 2i\sin(\alpha/2)\cos(\alpha/2)}{2i\sin(\alpha/2)\cos(\alpha/2) - 2\sin^2(\alpha/2)}$$

$$= \frac{2\cos(\alpha/2)[\cos(\alpha/2) + i\sin(\alpha/2)]}{2i\sin(\alpha/2)[\cos(\alpha/2) + i\sin(\alpha/2)]}$$

$$\Rightarrow \quad \frac{z_1}{z_2} = -\frac{1}{i}\cot\frac{\alpha}{2} \quad \Rightarrow \quad iz_1 = -\cot\frac{\alpha}{2}z_2$$

$\Rightarrow$ $iz_1 = Kz_2 \Rightarrow K = -\cot \alpha/2$

$\Rightarrow$ $\tan \alpha/2 = -1/K$

$$\tan\alpha = \frac{2\tan\alpha/2}{1 - \tan^2\alpha/2} \Rightarrow \frac{-2/K}{1 - 1/K^2} \Rightarrow \frac{-2K}{K^2 - 1}$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{2K}{1 - K^2}\right) = 2\tan^{-1}(K)$$

25. (c) : We have, $1 + \alpha + \alpha^2 + \ldots\ldots.. + \alpha^{n-1} = 0$

$$\Rightarrow \quad \sum_{r=0}^{n-1} \alpha^r = 0 \qquad \text{...(i)}$$

Now, $\displaystyle\sum_{r=0}^{n-1} |Z_1 + \alpha^r Z_2|^2 = \sum_{r=0}^{n-1} (Z_1 + \alpha^r Z_2)(\bar{Z}_1 + \bar{\alpha}^r \bar{Z}_2)$

$$= \sum_{r=0}^{n-1} |Z_1|^2 + Z_1\bar{Z}_2 \sum_{r=0}^{n-1} \bar{\alpha}^r + \bar{Z}_1 Z_2 \sum_{r=0}^{n-1} \alpha^r + \sum_{r=0}^{n-1} |Z_2|^2 |\alpha|^{2r}$$

$= n(|Z_1|^2 + |Z_2|^2)$, [Using (i) and $|\alpha| = 1$]

26. (b): Let E = Event that A gets six

$\therefore$ $P(E) = \dfrac{1}{6}$

F = Event that B gets six

$\therefore$ $P(F) = \dfrac{1}{6}$

$\therefore$ $P(B \text{ wins}) = P(\bar{E}\,F \text{ or } \bar{E}\,\bar{F}\,\bar{E}\,F \text{ or } \bar{E}\,\bar{F}\,\bar{E}\,\bar{F}\,\bar{E}F\ldots)$

(Since B can win the game in 2nd, 4th, 6th …… throw)

$$= \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3 \frac{1}{6} + \left(\frac{5}{6}\right)^5 \frac{1}{6} + \ldots\ldots$$

$$= \frac{5}{36}\left(1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots.\right) = \frac{5}{36}\left(\frac{1}{1 - \frac{25}{36}}\right) = \frac{5}{11}$$

27. (d): Let E_1 = The number less than or equal to 2 shows when a dice is thrown

E_2 = The event that six shows when a dice is thrown

$\therefore$ $P(E_1) = 2/6 = 1/3$ and $P(E_2) = 1/6$

E = The event that six turns up in the last throw = The event that E_1 happen in the all previous throws and E_2 happens in the last throw

$P(E) = P(E_1E_2 \text{ or } E_1E_1E_2 \text{ or } E_1E_1E_1E_2 \text{ or } \ldots..)$

$= P(E_1)\cdot P(E_2) + P(E_1)\cdot P(E_1)\cdot P(E_2) + P(E_1)\cdot P(E_1)\cdot P(E_1)\cdot P(E_2) + \ldots\ldots$

$$= \frac{1}{3}\cdot\frac{1}{6} + \left(\frac{1}{3}\right)^2 \cdot \frac{1}{6} + \left(\frac{1}{3}\right)^3 \cdot \frac{1}{6} + \ldots..$$

$$= \frac{1}{3}\cdot\frac{1}{6}\left[\frac{1}{1 - 1/3}\right] \Rightarrow P(E) = \frac{1}{12}$$

28. (a) : The given system of equations will be consistent

$$\text{if } \Delta = \begin{vmatrix} 1 & 1 & -3 \\ 1+\lambda & 2+\lambda & -8 \\ 1 & -(1+\lambda) & 2+\lambda \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 + 3C_1$, we get

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1+\lambda & 1 & -5+3\lambda \\ 1 & -2-\lambda & 5+\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $(5 + \lambda) + (2 + \lambda)(3\lambda - 5) = 0$

$\Rightarrow$ $5 + \lambda + 6\lambda - 10 + 3\lambda^2 - 5\lambda = 0$

$\Rightarrow$ $3\lambda^2 + 2\lambda - 5 = 0 \Rightarrow (3\lambda + 5)(\lambda - 1) = 0$

$\Rightarrow$ $\lambda = -5/3$ or $\lambda = 1$.

29. (b): For the system of equations having no solution, we must have

$$\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$$

$\Rightarrow$ $(k + 1)(k + 3) = 8k$ and $8(3k - 1) \neq 4k(k + 3)$

But $(k + 1)(k + 3) = 8k \Rightarrow k^2 + 4k + 3 = 8k$

$\Rightarrow$ $k^2 - 4k + 3 = 0 \Rightarrow (k - 1)(k - 3) = 0 \Rightarrow k = 1$ or 3

For $k = 1$, $8(3k - 1) = 16$ and $4k(k + 3) = 16$

$\therefore\ \ 8(3k-1) = 4k(k+3)$ for $k = 1$

For $k = 3$, $8(3k-1) = 64$ and $4k(k+3) = 72$

$\therefore\ \ 8(3k-1) \neq 4k(k+3)$ for $k = 3$.

Thus, there is just one value for which the given system of equations has no solution.

30. (b): $f(A) = I + A + A^2 + \dots\dots + A^{16}$

$$\text{Since, } A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{Similarly, } A^4 = A^5 = \dots\dots = A^{16} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \dots + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$$

31. (c): $\vec{a} \perp (\vec{b} + \vec{c}) \Rightarrow \vec{a} \cdot (\vec{b} + \vec{c}) = 0$

$\Rightarrow\ \ \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$

Also, $\vec{b} \cdot \vec{c} + \vec{b} \cdot \vec{a} = 0$ and $\vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} = 0$

Adding, $2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

Now, $|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c})$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$

$= 9 + 16 + 25 + 0 = 50$

$\therefore\ \ |\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$

32. (b): Lines are coplanar if $\begin{vmatrix} 1 & 1 & 1 \\ 2 & x_1 & x_2 \\ 3 & 4 & 5 \end{vmatrix} = 0$

$\Rightarrow\ \ 2x_1 - x_2 = 2$

Also, sum of roots $= \dfrac{-(x_2 + 2)}{x_1} = \dfrac{-2x_1}{x_1} = -2$

33. (a): Variance $= (\text{S.D.})^2 = \dfrac{1}{n}\Sigma x^2 - \left(\dfrac{\Sigma x}{n}\right)^2$,

$$\left(\because\ \ \bar{x} = \frac{\Sigma x}{n}\right)$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2 = \frac{n^2-1}{12}.$$

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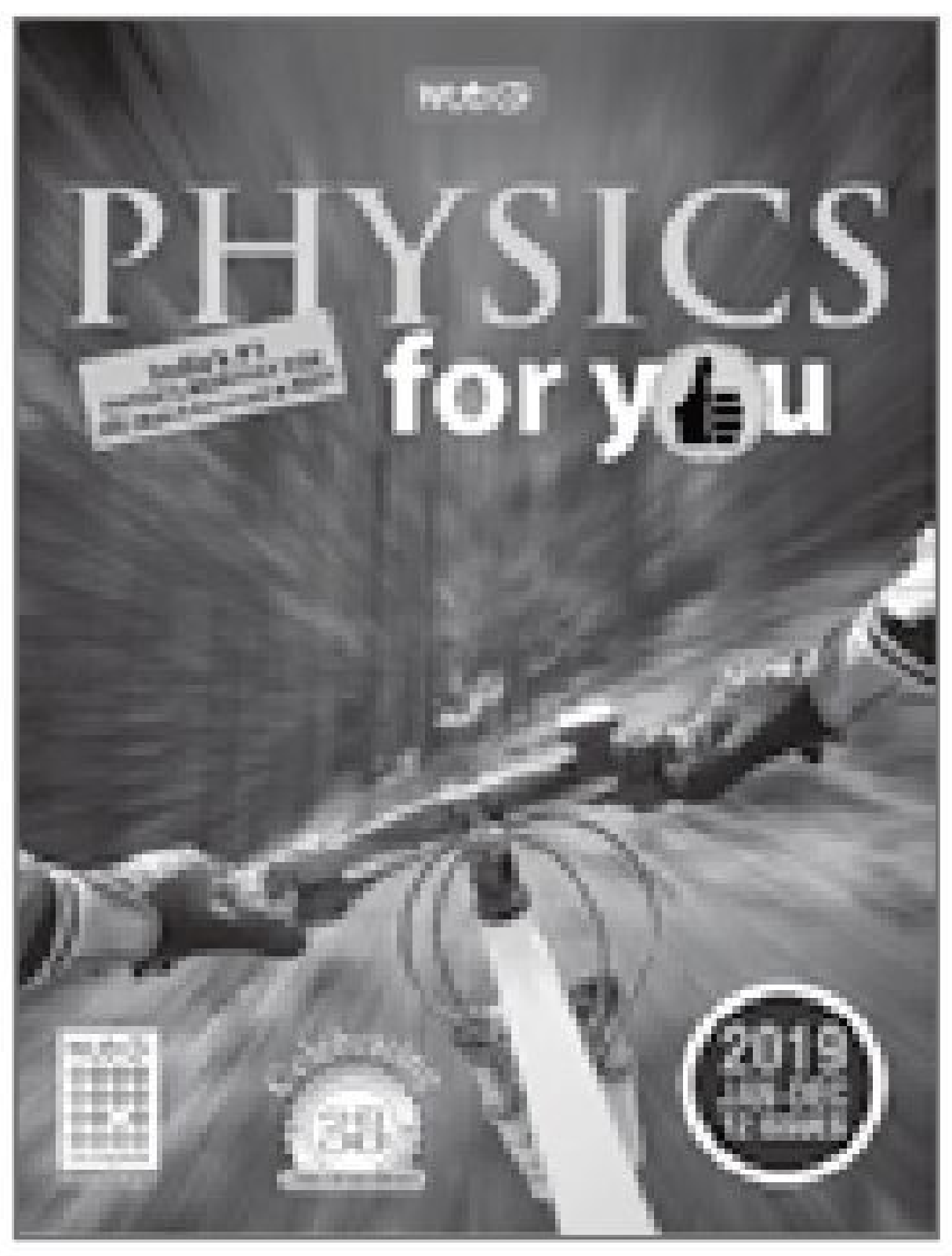
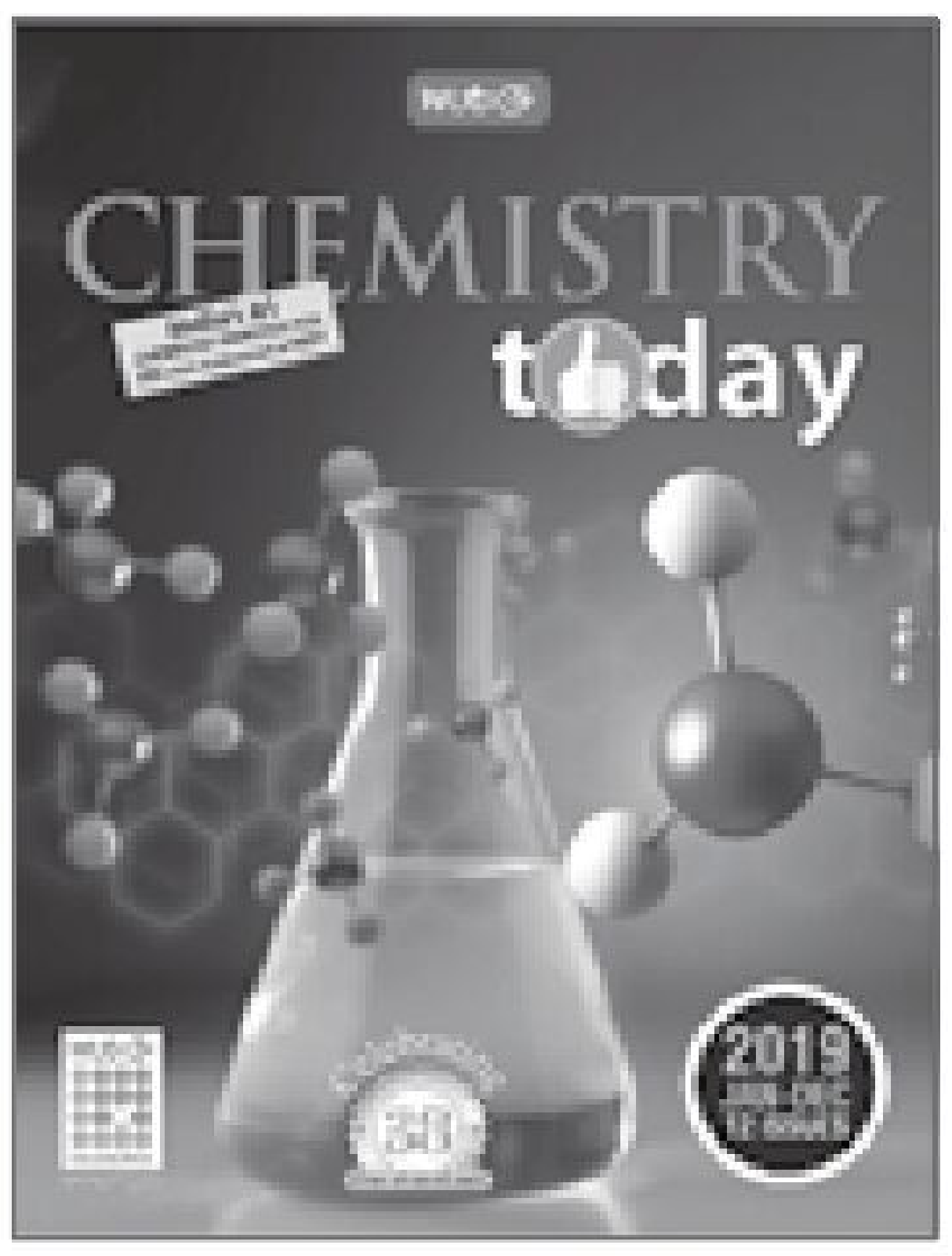
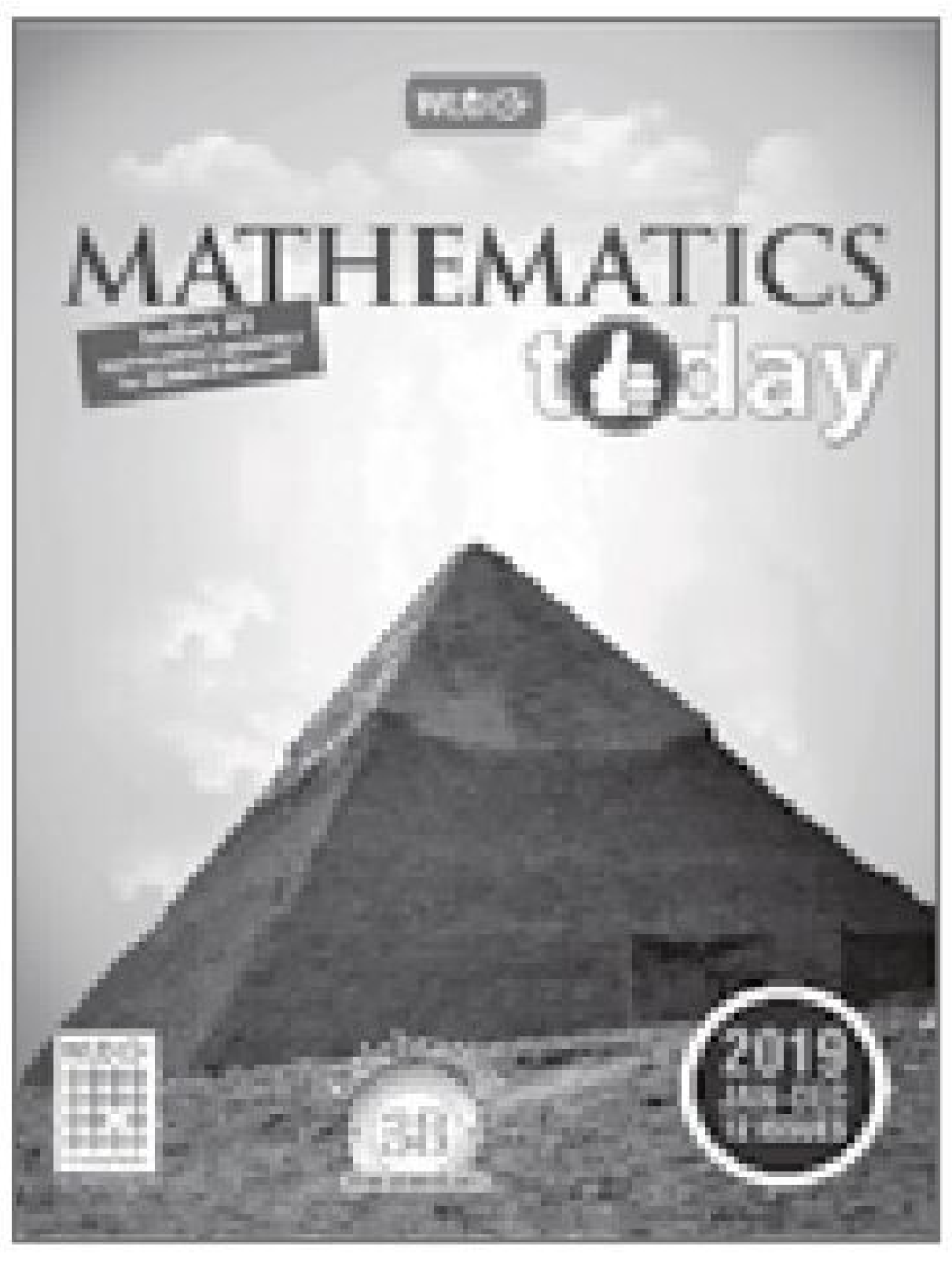
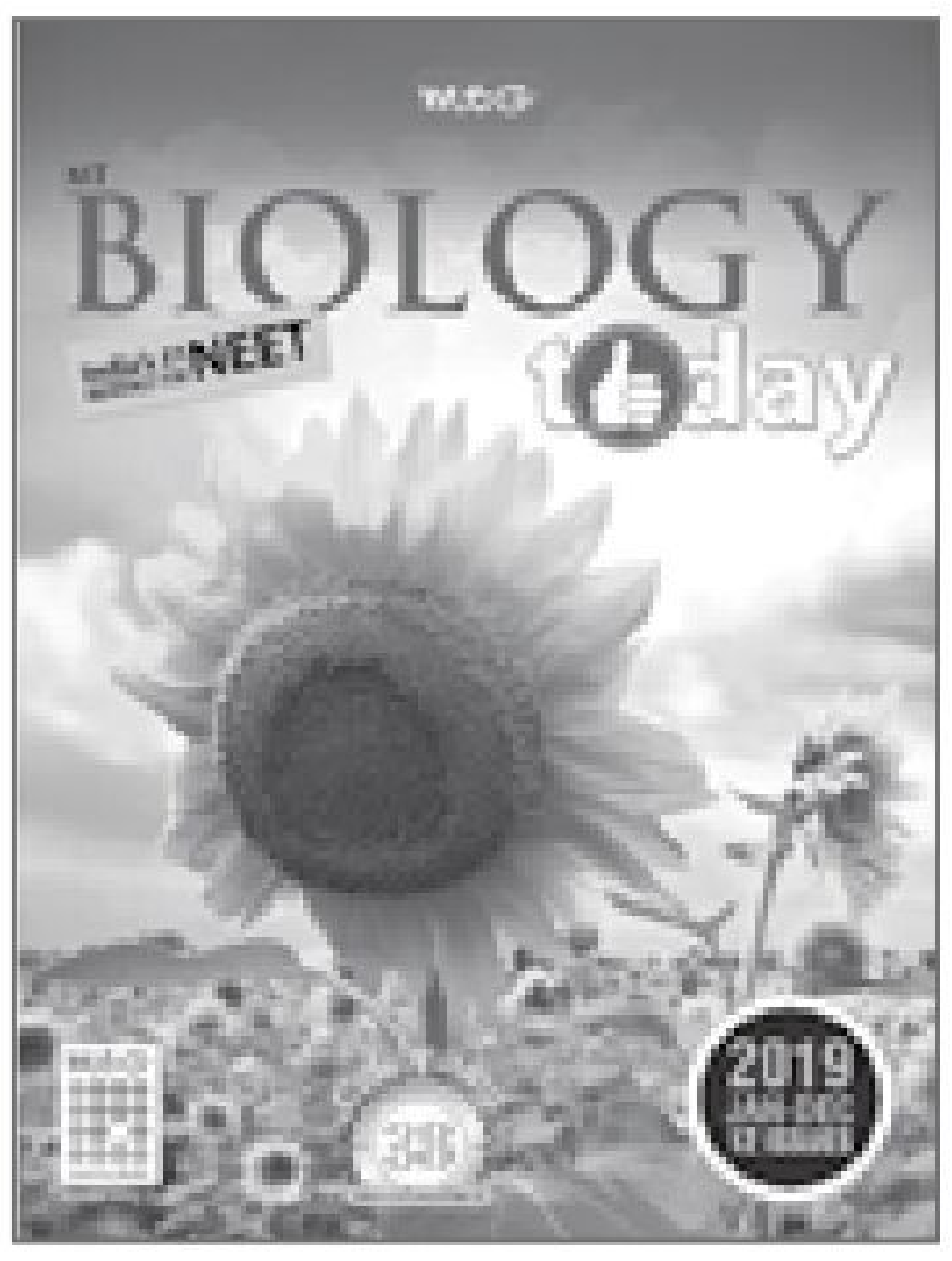

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CBSE warm-up!

CLASS-XI

Chapterwise practice questions for CBSE Exam as per the latest pattern

Series 2

Relations and Functions

General Instructions : Read the following instructions carefully.

(a) Q. No. 1 to 30 are objective type questions containing MCQs, Case Based MCQs, Assertion & Reason and Very Short Answer Type questions carrying 1 mark each.

(b) Q. No. 31 to 35 are Short Answer-I questions carrying 2 marks each.

(c) Q. No. 36 to 38 are Short Answer-II questions carrying 3 marks each.

(d) Q. No. 39 to 40 are Long Answer questions carrying 5 marks each.

MCQs

1. The range of the function $f(x)=\dfrac{x-2}{2-x}, x \neq 2$ is

(a) R (b) $R-\{1\}$

(c) $\{-1\}$ (d) $R-\{-1\}$

2. The domain of the function $f=\{(1, 3), (3, 5), (2, 6)\}$ is

(a) 1, 3 and 2 (b) {1, 3, 2}

(c) {3, 5, 6} (d) 3, 5 and 6

3. Let $f(x)=\dfrac{x-1}{x+1}$, then $f(f(x))$ is

(a) $\dfrac{1}{x}$ (b) $-\dfrac{1}{x}$ (c) $\dfrac{1}{x+1}$ (d) $\dfrac{1}{x-1}$

4. If $f(x) = e^x$, then $\dfrac{f(a)}{f(-b)}$ is equal to

(a) $f(a+b)$ (b) $f(a-b)$

(c) $f(-a+b)$ (d) $f(-a-b)$

5. Let $n(A) = 6$ and $n(B) = p$. Then, the total number of non-empty relations that can be defined from A to B is

(a) 6^p (b) $n^p - 1$

(c) $6p - 1$ (d) $2^{6p} - 1$

6. If $f(x) = 9x - x^2, x \in R$, then $f(a+1) - f(a-1)$ is equal to

(a) $4(4-a)$ (b) $2(9-2a)$

(c) $4(2+a)$ (d) $2(4+a)$

7. The domain of the function $f(x)=\dfrac{1}{\sqrt{|x|-x}}$ is

(a) $(-\infty, 0)$ (b) $(-\infty, \infty)-\{0\}$

(c) $(-\infty, \infty)$ (d) $(0, \infty)$

8. Suppose that the number of elements in set A is p, the number of elements in set B is q and the number of elements in $A \times B$ is 7. Then $p^2 + q^2 =$

(a) 42 (b) 49

(c) 50 (d) 51

9. If $A = \{1, 2, 3\}$, $B = \{3, 8\}$, then $(A \cup B) \times (A \cap B)$ is equal to

(a) $\{(3, 1), (3, 2), (3, 3), (3, 8)\}$

(b) $\{(1, 3), (2, 3), (3, 3), (8, 3)\}$

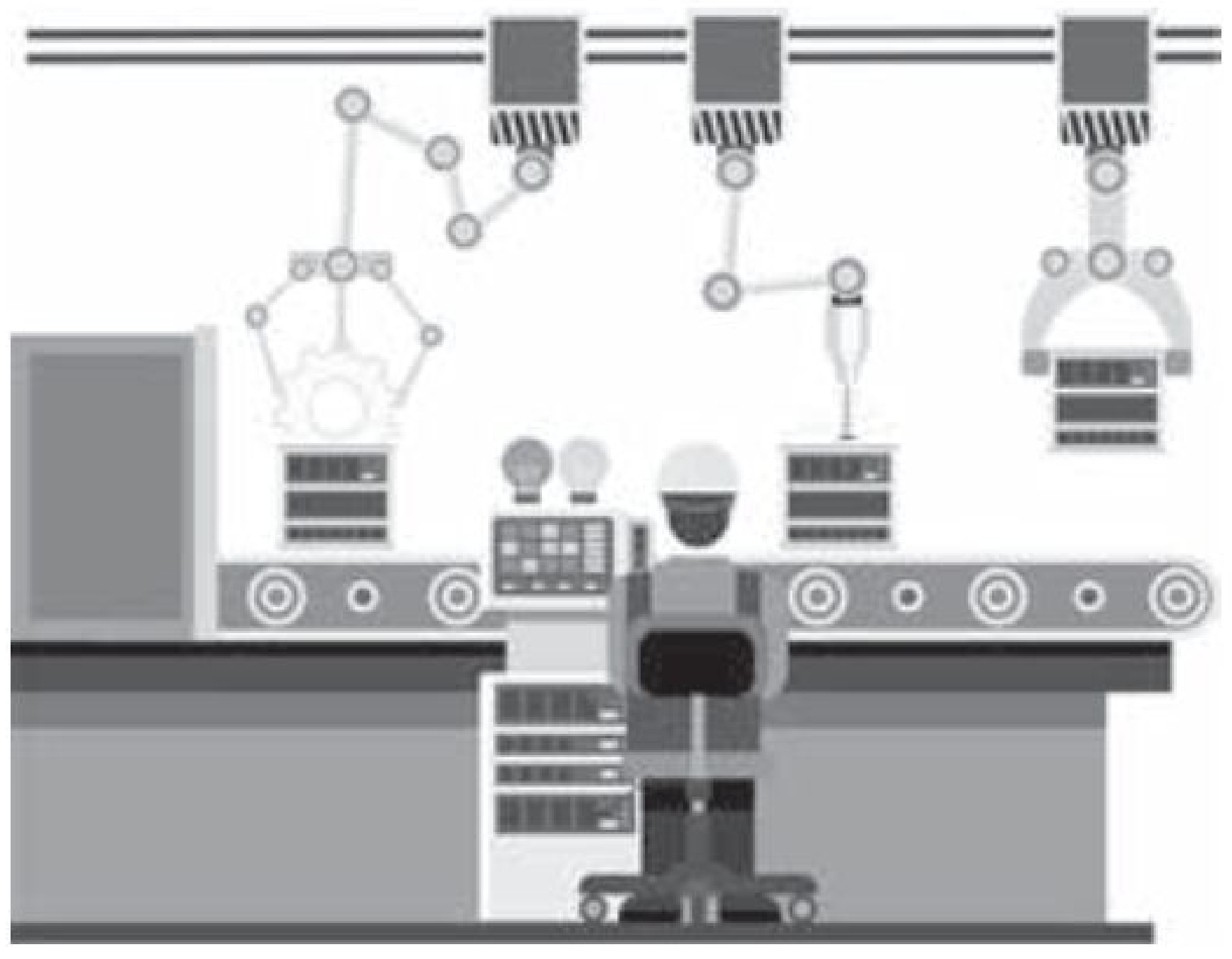
(c) $\{(1, 2), (2, 2), (3, 3), (8, 8)\}$

(d) $\{(8, 3), (8, 2), (8, 1), (8, 8)\}$.

10. Let $A = \{x : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$ then $A \times (B \cap C)$ is
(a) $\{(2, 4), (3, 4)\}$
(b) $\{(4, 2), (4, 3)\}$
(c) $\{(2, 4), (3, 4), (4, 4)\}$
(d) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

CASE BASED

Case-1 : One day Rohit, who is Class XI student visited his father's factory, where he observed various kind of machines. He visualize a function as a machine, which produce new element $y = f(x)$; if an element x was put in it. He explore this concept and try to find the answer of following questions.



11. What will be the output of $x = 4.5$, if the function is $f(x) = [x]$, where $[\cdot]$ denote greatest integer function?
(a) 3 (b) 4 (c) 5 (d) 6

12. What will be the output of $x = -39$, if function is $f(x) = \text{sgn}(x)$?
(a) 1 (b) 0 (c) -1 (d) i

13. If ordered pairs in a relation are (2, 3), (2, 4), (3, 4), (3, 5) can we say that it can be visualize as a function?
(a) Yes
(b) No
(c) May or may not be
(d) Data inadequate

14. If the function visualized as machine is $f : R^+ \to R$ defined by $f(x) = \sqrt{x}$, then domain of function is
(a) R^+ (b) R^- (c) R (d) $[-1, 1]$

15. Find the range of function defined in question 14.
(a) R^- (b) R^+ (c) R (d) $[-1, 1]$

Case-2 : In a school, Inter school cricket tournament is going to held. So, the class teacher told Ashish, who is the monitor of the Class XI, to collect the name of students interested in cricket from Class XI and XII. Ashish wrote the name of Class XI students interested in cricket match as set A = {Anil, Sonu, Mintu, Pinku}. Also, students interested in cricket match of Class XII is written as set B = {Anjanay, Ankur, Harish}. During this activity class teacher was doing discussion on relations and functions. He explored these two sets for various type of relations and functions. Based on the above information, answer the following questions.

CRICKET
TOURNAMENT

16. Number of elements in $A \times B$ is
(a) 4 (b) 3 (c) 12 (d) 16

17. Number of possible relations from set A to set B is
(a) 2^4 (b) 2^{12} (c) 2^3 (d) 2^{10}

18. Number of relations on set A is
(a) 2^{16} (b) 2^{12} (c) 2^{10} (d) 10

19. Which of the following is a function from B to A?
(a) {(Anjanay, Sonu), (Harish, Mintu)}
(b) {(Anjanay, Pinku), (Harish, Pinku), (Harish, Sonu)}
(c) {(Anjanay, Anil), (Ankur, Mintu), (Harish, Sonu)}
(d) {(Anjanay, Pinku), (Anjanay, Sonu), (Anjanay, Mintu)}

20. Number of possible functions from set A to set B is
(a) 3^5 (b) 4^3 (c) 5^3 (d) 3^4

ASSERTION & REASON

Directions (Q.No. 21-25) : In the following questions, a statement of assertion (Statement-I) is followed by a statement of reason (Statement-II). Mark the correct choice as :

(a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.
(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.
(c) If Statement-I is true but Statement-II is false.
(d) If Statement-I is false and Statement-II is true.

21. **Statement-I :** If $f(x) = 2x^2 + bx + c$ and $f(0) = 3$ and $f(2) = 1$, then $f(1)$ is equal to zero.
Statement-II : If $f : X \to R$ and $g : X \to R$ be two real functions, where $X \subset R$, then $(f + g)(x) = f(x) + g(x) \forall\ x \in X$.

22. **Statement-I :** The domain of function $f(x) = \dfrac{|x+3|}{x+3}$ is $R - \{-3\}$.

Statement-II : Domain of modulus function is set of all real number but range is the set of all non-negative real numbers.

23. **Statement-I :** Let $f(x) = 1 + x$, $g(x) = x^2 + x + 1$, then $(f + g)(x)$ at $x = 0$ is equal to 9.

Statement-II : $f : X \to R$ and $g : X \to R$ be two real valued functions, then $\dfrac{f}{g} : X \to R$ defined as $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, provided $g(x) \neq 0 \ \forall \ x \in X$.

24. **Statement-I :** If $A = \{x, y, z\}$ and $B = \{3, 4\}$, then number of relations from A to B is 2^5.

Statement-II : Number of relations from A to B is $2^{n(A) \times n(B)}$.

25. **Statement-I :** The domain of the relation R defined by $R = \{(x, x + 5) : x \in (0, 1, 2, 3, 4, 5)\}$ is $\{5, 6, 7, 8, 9, 10\}$.

Statement-II : The collection of first elements of all ordered pairs is called domain.

VSA

26. Let $R = \{(a, a^3) : a$ is a prime number less than $5\}$ be a relation. Find the range of R.

27. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and if $R = \{(x, y) : y$ is half of x; $x, y \in A\}$ is a relation on A, then write R as a set of ordered pairs.

28. If $f\colon R \to R$ is defined by $f(x) = 2^x$, then find range of f.

29. If $f(x) = x + 1$ and $g(x) = 2x - 3$ be two real functions, find the value of $(f^2 - 3g)(x)$.

30. If $A = \{1, 2\}$, then form the set $A \times A \times A$.

SA - I

31. Find $f(x) + f(1 - x)$, if $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$.

32. Let $f : R \to R$ be defined by $f(x) = \sin x$ and $g : R \to R$ be defined by $g(x) = x$, find $f + g$, $f - g$, $f \cdot g$, $\dfrac{f}{g}$ and αf, $\alpha \in R$.

33. Find the domain of the function $f(x)$ defined by

$$f(x) = \sqrt{4 - x} + \frac{1}{\sqrt{x^2 - 1}}.$$

34. Let $f : Z \to Z$, $g : Z \to Z$ be functions defined by $f = \{(n, n^2) : n \in Z\}$ and $g = \{(n, |n|^2) : n \in Z\}$. Show that $f = g$.

35. If $P = \{a, b\}$ and $Q = \{x, y, z\}$, then show that $P \times Q \neq Q \times P$.

SA - II

36. If for non-zero x, $a\,f(x) + bf\left(\dfrac{1}{x}\right) = \dfrac{1}{x} - 5$, where $a \neq b$, then find $f(x)$.

37. If $A = \{x : x \in W, x < 3\}$, $B = \{x : x \in N, 2 \leq x < 4\}$ and $C = \{3, 4\}$, then verify that $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

38. Let R be a relation from Q to Q defined by $R = \{(a, b) : a, b \in Q$ and $a - b \in Z\}$. Show that
(a) $(a, a) \in R$ for all $a \in Q$.
(b) $(a, b) \in R$ implies that $(b, a) \in R$
(c) $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, c) \in R$.

LA

39. Let f, g be two functions defined by $f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9 - x^2}$, then find

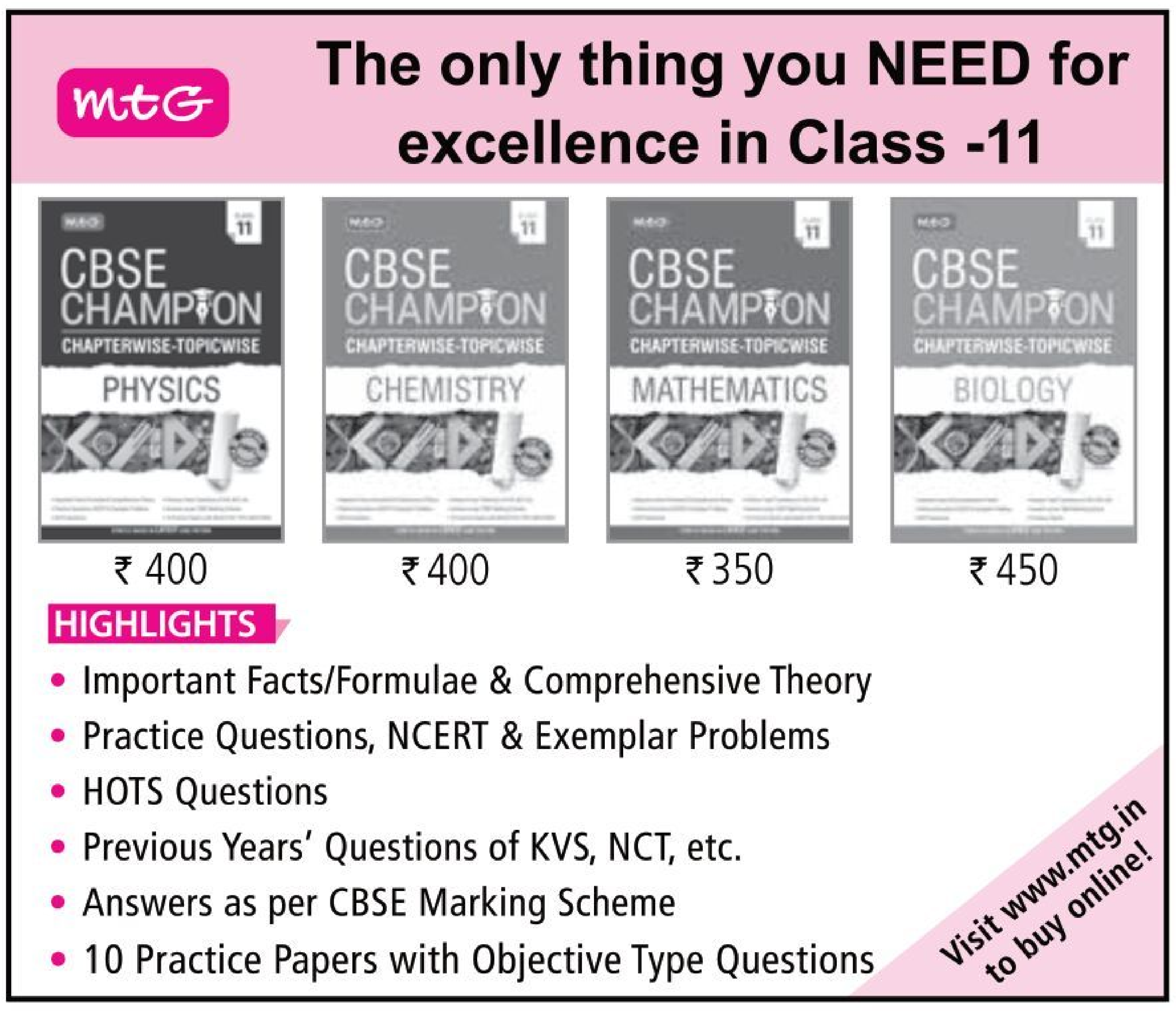
(i) $\dfrac{f}{g}$ (ii) $\dfrac{g}{f}$ (iii) $2f - \sqrt{5}g$

(iv) $f^2 + 7f$ (v) $\dfrac{3}{g}$.

40. The relations f and g are defined by

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases} \text{ and } g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10. \end{cases}$$

Show that f is a function and g is not a function.



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SOLUTIONS

1. **(c) :** We have, $f(x)=\dfrac{x-2}{2-x}=-\dfrac{x-2}{x-2}=-1$ if $x\neq 2$

$\therefore$ Range of $f = \{-1\}$

2. **(b) :** Domain of f is the set of those points on which f is defined. Here, $f(1) = 3$, $f(3) = 5$ and $f(2) = 6$. So, $D_f = \{1, 2, 3\}$

3. **(b) :** $f(f(x)) = f\left(\dfrac{x-1}{x+1}\right) = \dfrac{\dfrac{x-1}{x+1}-1}{\dfrac{x-1}{x+1}+1} = \dfrac{-2}{2x} = \dfrac{-1}{x}$

4. **(a) :** $\dfrac{f(a)}{f(-b)} = \dfrac{e^a}{e^{-b}} = e^a e^b = e^{a+b} = f(a+b)$

5. **(d) :** Given $n(A) = 6$ and $n(B) = p$

$\therefore$ Total number of relations from A to $B = 2^{6p}$

$\therefore$ Total number of non-empty relations from A to $B = 2^{6p} - 1$

6. **(b) :** We have, $f(a + 1) - f(a - 1)$
$= [9(a + 1) - (a + 1)^2] - [9(a - 1) - (a - 1)^2]$
$= 18 - 4a = 2(9 - 2a)$

7. **(a) :** We have, $f(x)=\dfrac{1}{\sqrt{|x|-x}}$

Now, $|x| - x > 0 \Rightarrow |x| > x$
Thus, x must be negative
$\Rightarrow x \in (-\infty, 0)$

8. **(c) :** $n(A) = p$, $n(B) = q$, $n(A \times B) = pq = 7$
So possible values of p, q are 7, 1
$\Rightarrow p^2 + q^2 = (7)^2 + (1)^2 = 50.$

9. **(b) :** $A \cup B = \{1, 2, 3, 8\}$ and $A \cap B = \{3\}$
$\therefore (A \cup B) \times (A \cap B) = \{(1, 3), (2, 3), (3, 3), (8, 3)\}$

10. **(a) :** $A = \{x : x^2 - 5x + 6 = 0\}$
$x^2 - 5x + 6 = 0 \Rightarrow (x - 3)(x - 2) = 0 \Rightarrow x = 3, 2$
$\therefore A = \{3, 2\}, B = \{2, 4\}, C = \{4, 5\}$
Now, $B \cap C = \{4\}$
$\Rightarrow A \times (B \cap C) = \{3, 2\}\times\{4\} = \{(3, 4), (2, 4)\}.$

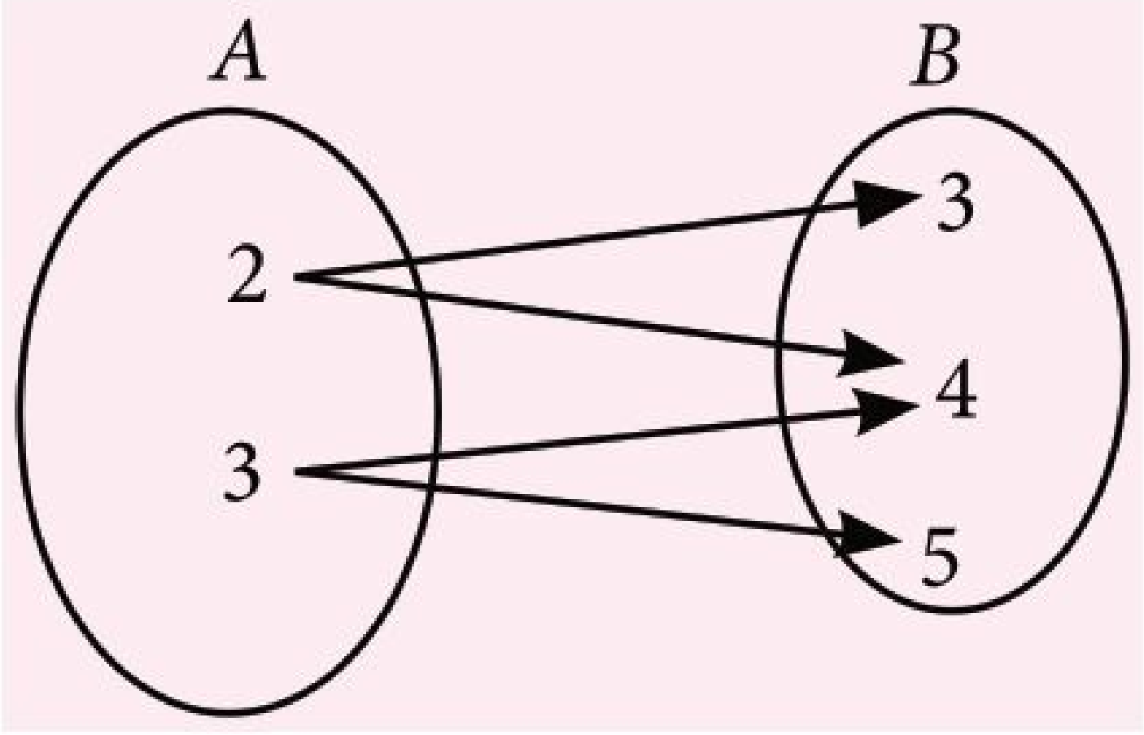
11. **(b) :** We have, $f(x) = [x]$
At $x = 4.5$; $f(x) = [4.5] = 4$

12. **(c) :** We have, $f(x) = \text{sgn}(x)$

and $\text{sgn}(x) = \begin{cases} -1, & x<0 \\ 0, & x=0 \\ 1, & x>0 \end{cases}$

At $x = -39$, $f(x) = -1$

13. **(b) :** Let $f(x) = \{(2, 3), (2, 4), (3, 4), (3, 5)\}$



Since, each element in A is associated with two elements in B.
$\therefore$ f is not a function.

14. **(a)** 15. **(b)**

16. **(c) :** $n(A) = 4$, $n(B) = 3$
$\therefore n(A \times B) = 4 \times 3 = 12$

17. **(b) :** Number of relations from set A to set B $= 2^{4\times 3} = 2^{12}$

18. **(a) :** Number of relations on set $A = 2^{4^2} = 2^{16}$

19. **(c) :** Since, each element in B is associated with a unique element in A.
$\therefore$ Option (c) can be defined as a function.

20. **(d) :** Number of possible functions from set A to set $B = 3^4$

21. **(b) :** Clearly, Statement-II is true.
Now, $f(x) = 2x^2 + bx + c$
$f(0) = 3 \Rightarrow c = 3$
$f(2) = 1 \Rightarrow 8 + 2b + c = 1 \Rightarrow 2b + 3 = -7 \Rightarrow b = -5$
$\therefore f(1) = 2(1)^2 + (-5)(1) + 3 = 0$
$\therefore$ Statement-I is true.
Thus, both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.

22. **(a) :** Clearly, Statement-II is true.

Now $f(x)=\dfrac{|x+3|}{x+3}$

Here $f(x)$ is defined only when $x + 3 \neq 0$
i.e., $x \neq -3$
$\therefore$ Domain $(f) = R - \{-3\}$
$\therefore$ Statement-I is true.
Thus, both Statement-I and Statement-II are true and Statement-II is correct explanation of Statement-I.

23. **(d) :** Clearly, Statement-II is true.
Now, $f(x) = 1 + x$, $g(x) = x^2 + x + 1$
$(f + g)(x) = f(x) + g(x) = 1 + x + x^2 + x + 1 = x^2 + 2x + 2$
$\therefore (f + g)(0) = 2 \neq 9$
$\therefore$ Statement-I is false.
$\therefore$ Statement-I is false, but Statement-II is true.

24. (d) : We have,
$A = \{x, y, z\}, B = \{3, 4\} \Rightarrow n(A) = 3, n(B) = 2$
$\therefore \quad n(A \times B) = n(A) \times n(B) = 6$
Therefore, the number of subsets of $A \times B$ is 2^6.
So, the number of relations from A to B is 2^6.
$\therefore$ Statement-I is false but Statement-II is true.

25. (d) : We have, $R = \{(x, x + 5) : x \in (0, 1, 2, 3, 4, 5)\}$
$R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$
$\therefore$ Domain of $R = \{0, 1, 2, 3, 4, 5\}$

26. We have, $R = \{(a, a^3) : a$ is prime number less than $5\}$
$\Rightarrow \quad R = \{(a, a^3) : a = 2, 3\}$
$\Rightarrow \quad R = \{(2, 8), (3, 27)\}$
$\therefore$ Range of $R = \{8, 27\}$

27. We have, $R = \{(x, y) : y$ is half of $x\,;\, x, y \in A\}$
$\therefore \quad R = \{(2, 1), (4, 2), (6, 3), (8, 4)\}$

28. Given $f : R \to R$ is defined by $f(x) = 2^x$.
Since $D_f = R$ and for all $x \in R$, $2^x > 0$
$\therefore$ Range of $f = (0, \infty)$.

29. $(f^2 - 3g)(x) = (f^2)(x) - (3g)(x) = (f(x))^2 - 3g(x)$
$= (x + 1)^2 - 3(2x - 3) = x^2 + 2x + 1 - 6x + 9$
$= x^2 - 4x + 10$, for all $x \in R$.

30. We have, $A = \{1, 2\}$
$\therefore \quad A \times A = \{1, 2\} \times \{1, 2\} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$
Again, $(A \times A) \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\} \times \{1, 2\}$
$\Rightarrow \quad A \times A \times A = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}$

31. Given, $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$ …(i)

$$\therefore \quad f(1-x) = \frac{a^{1-x}}{a^{1-x} + \sqrt{a}} = \frac{a \cdot a^{-x}}{a \cdot a^{-x} + \sqrt{a}}$$

$$= \frac{a}{a + \sqrt{a} \cdot a^x} = \frac{\sqrt{a}}{\sqrt{a} + a^x} \qquad \text{…(ii)}$$

Adding (i) and (ii), we get

$$f(x) + f(1 - x) = \frac{a^x}{a^x + \sqrt{a}} + \frac{\sqrt{a}}{\sqrt{a} + a^x} = \frac{a^x + \sqrt{a}}{a^x + \sqrt{a}} = 1$$

32. Given, $f(x) = \sin x$ and $g(x) = x$
Now, $f + g : R \to R$, $(f + g)(x) = f(x) + g(x) = \sin x + x$
$(f - g) : R \to R$, $(f - g)(x) = f(x) - g(x) = \sin x - x$
$(f \cdot g) : R \to R$, $(f \cdot g)(x) = f(x) \cdot g(x) = (\sin x)x = x \sin x$
$\dfrac{f}{g} : R \to R$, $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sin x}{x}, x \neq 0$
$\alpha f : R \to R$, $(\alpha f)(x) = \alpha f(x) = \alpha \sin x$

33. Clearly, $f(x)$ is defined for $4 - x \geq 0$ and $x^2 - 1 > 0$
$\Rightarrow \quad x - 4 \leq 0$ and $(x - 1)(x + 1) > 0$
$\Rightarrow \quad x \leq 4$ and $(x < -1$ or $x > 1) \Rightarrow x \in (-\infty, -1) \cup (1, 4]$.
Hence, Domain of $f(x) = (-\infty, -1) \cup (1, 4]$.

34. Given, domain of $f = Z$ and domain of $g = Z$
Hence, domain f = domain $g = Z$ …(i)
Also, $f(n) = n^2$, for all $n \in Z$ and $g(n) = |n|^2 = n^2$ for all $n \in Z$
Hence, $f(n) = g(n)$ for all $n \in Z$ …(ii)
From (i) and (ii), we have $f = g$.

35. We have, $P = \{a, b\}$ and $Q = \{x, y, z\}$
Now, $P \times Q = \{(a, x), (a, y), (a, z), (b, x), (b, y), (b, z)\}$
$Q \times P = \{(x, a), (x, b), (y, a), (y, b), (z, a), (z, b)\}$
$\therefore \quad P \times Q \neq Q \times P$.

36. We have, $a\,f(x) + b\,f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} - 5$ …(i)

Replacing x by $\dfrac{1}{x}$ we get,

$$a\,f\left(\frac{1}{x}\right) + b\,f(x) = x - 5 \qquad \text{…(ii)}$$

Multiplying (i) by a and (ii) by b, we get

$$a^2 f(x) + abf\left(\frac{1}{x}\right) = \frac{a}{x} - 5a \qquad \text{…(iii)}$$

$$abf\left(\frac{1}{x}\right) + b^2 f(x) = bx - 5b \qquad \text{…(iv)}$$

Subtracting (iv) from (iii), we get

$$(a^2 - b^2)\,f(x) = \left(\frac{a}{x} - bx\right) - 5(a - b)$$

$$\Rightarrow \quad f(x) = \frac{1}{a^2 - b^2}\left\{\frac{a}{x} - bx\right\} - \frac{5}{a + b}$$

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37. Given, $A = \{x : x \in W, x < 3\} = \{0, 1, 2\}$

$B = \{x : x \in N, 2 \le x < 4\} = \{2, 3\}$ and $C = \{3, 4\}$

$\Rightarrow \quad A \cup B = \{0, 1, 2, 3\}$.

$\therefore \quad (A \cup B) \times C = \{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$ ...(i)

Also, $A \times C = \{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4)\}$

Also, $B \times C = \{(2, 3), (2, 4), (3, 3), (3, 4)\}$.

$\therefore \quad (A \times C) \cup (B \times C) = \{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$ ...(ii)

From (i) and (ii), we get

$(A \cup B) \times C = (A \times C) \cup (B \times C)$.

38. We have, $R = \{(a, b) : a, b \in Q$ and $a - b \in Z\}$

(a) Let $a \in Q$. Then $a - a = 0 \in Z$.

$\therefore \quad (a, a) \in R \ \forall \ a \in Q$.

(b) $(a, b) \in R \ \Rightarrow \ (a - b) \in Z$

$\Rightarrow \ -(a - b) \in Z \ (\because (-1) \in Z) \ \Rightarrow \ b - a \in Z$

$\Rightarrow \quad (b, a) \in R$

Thus, $(a, b) \in R \ \Rightarrow \ (b, a) \in R$

(c) Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow \quad a - b \in Z$ and $b - c \in Z \Rightarrow \{(a - b) + (b - c)\} \in Z$

$\Rightarrow \quad (a - c) \in Z \ \Rightarrow \ (a, c) \in R$

Thus, $(a, b) \in R$ and $(b, c) \in R \ \Rightarrow \ (a, c) \in R$

39. Given, $f(x) = \sqrt{x+1}, \ g(x) = \sqrt{9 - x^2}$

$\Rightarrow \quad D_f = [-1, \infty)$ and $D_g = [-3, 3]$.

Let $D = D_f \cap D_g = [-1, \infty) \cap [-3, 3] = [-1, 3] \ne \phi$, then

(i) $$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+1}}{\sqrt{9-x^2}} = \sqrt{\frac{x+1}{9-x^2}}$$

with domain D_1, where $D_1 = D$ except where $g(x) = 0 \Rightarrow D_1 = [-1, 3)$.

(ii) $$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{9-x^2}}{\sqrt{x+1}} = \sqrt{\frac{9-x^2}{x+1}}$$

with domain D_2, where $D_2 = D$ except where $f(x) = 0 \Rightarrow D_2 = (-1, 3]$.

(iii) $(2f - \sqrt{5}g)(x) = 2f(x) - \sqrt{5}g(x)$

$= 2\sqrt{x+1} - \sqrt{5}\sqrt{9-x^2}$ with domain $[-1, 3]$.

(iv) $(f^2 + 7f)\ (x) = (f(x))^2 + 7f(x) = x + 1 + 7\sqrt{x+1}$ with domain $[-1, \infty)$.

(v) $$\left(\frac{3}{g}\right)(x) = \frac{3}{g(x)} = \frac{3}{\sqrt{9-x^2}}$$ with domain D_3, where

$D_3 = D_g$ except where $g(x) = 0 \Rightarrow D_3 = (-3, 3)$.

40. Given $f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10. \end{cases}$

Note that $D_f = [0, 10]$.

For $0 \le x \le 3$, $f(x) = x^2$, so $f(3) = 3^2 = 9$;

and for $3 \le x \le 10$, $f(x) = 3x$, so $f(3) = 3 \times 3 = 9$.

Thus, for each element of the domain of f has a unique image. Therefore, f is a function.

Now, $g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$.

Note that $D_g = [0, 10]$.

For $0 \le x \le 2$, $g(x) = x^2$, so $g(2) = 2^2 = 4$;

and for $2 \le x \le 10$, $g(x) = 3x$, so $g(2) = 3 \times 2 = 6$.

Thus, the element 2 of the domain of g has two different images. Therefore, g is not a function.

◆ ◆

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

		1	3		9	8							2	6			8	5		
	3					2	6	9					3		5	2			4	
8			2			3	1					8				9		7	2	3
6	4		7					1				3		7			2			6
3	1			9			4							8			6		5	2
	9	8			1		3					2		5	3				8	
		3	1		8	4		7	3	5		6					5		3	
		4		3				6	9				8		2	6			7	
9				2		1				2			7				3	2		
						8	3			9			5							
							7	9			5		2							
						2			8	4		3								
4			7			9	1		5		4	7			9		2		6	
	8	9	4					5		8	9	2				3	6		7	
		1		9	5	7					3			5	7		8			3
	4			1		8	9					8		3		6				7
	9				8		7	4				6	7				9	2		
8			9			1							2			7	3			6
		7	8			4								7		2		6		
5				4	9								8		6				9	
2			1					9						6			7	3		

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MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 2 : Complex Numbers and Quadratic Equations

Time Taken : 60 Min.

Only One Option Correct Type

1. If $a = \cos\theta + i\sin\theta$ and $b = \cos\phi + i\sin\phi$, then

(a) $\cos(\theta+\phi) = \left(ab - \frac{1}{ab}\right)$

(b) $\cos(\theta+\phi) = \frac{1}{2}\left(\frac{a}{b} + \frac{b}{a}\right)$

(c) $\cos(\theta+\phi) = \frac{1}{2}\left(ab + \frac{1}{ab}\right)$

(d) $\cos(\theta+\phi) = \left(\frac{a^2 - b^2}{ab}\right)$

2. If $-3 < \frac{x^2 + mx + 1}{x^2 + x + 1} < 3$ for all real x, then

(a) $m < -1$ (b) $-1 < m \le 7$

(c) $-1 < m < 5$ (d) $m \ge 6$

3. The value of the expression $1\cdot(2-\omega)(2-\omega^2) + 2\cdot(3-\omega)(3-\omega^2) + \dots\dots\dots + (n-1)\cdot(n-\omega)(n-\omega^2)$, where ω is an imaginary cube root of unity, is

(a) $\frac{1}{2}(n-1)n(n^2+3n+4)$

(b) $\frac{1}{4}(n-1)n(n^2+3n+4)$

(c) $\frac{1}{2}(n+1)n(n^2+3n+4)$

(d) $\frac{1}{4}(n+1)n(n^2+3n+4)$

4. If $\frac{3}{2+\cos\theta + i\sin\theta} = a + ib$, then $(a-2)^2 + b^2$ equals

(a) 0 (b) 1 (c) -1 (d) 2

5. If x_1, x_2 are the roots of $ax^2 + bx + c = 0$ and $x_1 + d$, $x_2 + d$ are the roots of $px^2 + qx + r = 0$, where $d \ne 0$, then

(a) $b^2 - 4ac = q^2 + 4qr$ (b) $\frac{a}{p+d} = \frac{b}{q+d} = \frac{c}{d+r}$

(c) $d = \frac{bp - aq}{2ap}$ (d) $a + d = p$

6. If $1, \omega, \omega^2, \dots, \omega^{n-1}$ are n, n^{th} roots of unity, then the value of $(9-\omega)(9-\omega^2)\dots(9-\omega^{n-1})$ will be

(a) n (b) 0

(c) $\frac{9^n - 1}{8}$ (d) $\frac{9^n + 1}{8}$

One or More Than One Option(s) Correct Type

7. If $y = \log_{7-a}(4x^2 + 2x + a + 3)$ is defined $\forall\, x \in R$, then possible integral value(s) of a is(are)

(a) -3 (b) -2 (c) 4 (d) 5

8. If p, q and r are positive real numbers and are in A.P., the roots of the quadratic equation $px^2 + qx + r = 0$ are real for

(a) $\left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$ (b) $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$

(c) All p and all r (d) No p and no r

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9. Let the complex number z of the form $x + iy$ satisfy $\arg\left(\frac{3z-6-3i}{2z-8-6i}\right)=\frac{\pi}{4}$ and $|z - 3 + i| = 3$. Then the ordered pair(s) (x, y) is/are

(a) $\left(4-\frac{4}{\sqrt{5}},1+\frac{2}{\sqrt{5}}\right)$ (b) $\left(4+\frac{4}{\sqrt{5}},1-\frac{2}{\sqrt{5}}\right)$

(c) $(6, -1)$ (d) $(0, -1)$

10. The equation $x^{[\log_3 x)^2-(9/2)\log_3 x+5]}=3\sqrt{3}$ has

(a) at least one real solution

(b) exactly three real solutions

(c) exactly one irrational solution

(d) complex roots

11. If $\omega \neq 1$ is a cube root of unity, then sum of the series, $S = 1 + 2\omega + 3\omega^2 + ... + 3n\,\omega^{3n-1}$ $(n \in N)$ is

(a) $\frac{3n}{\omega-1}$ (b) $n(\omega^2 - 1)$

(c) 0 (d) None of these

12. If $x \in R$, then range of $\frac{x^2-2x+3}{x^2-2x-8}$ is

(a) $\left[\frac{-2}{9},1\right)$ (b) $\left[\frac{-2}{9},1\right]$

(c) $\left(-\infty,-\frac{2}{9}\right]\cup(1,\infty)$ (d) None of these

13. If $z_0=\frac{1+i}{2}$, then the value of the product $(1+z_0)(1+z_0^2)(1+z_0^{2^2})\ldots\ldots(1+z_0^{2^n})$ must be

(a) $2^{2^{n+1}}$

(b) $(1+i)\left(1-\frac{1}{2^{2^n}}\right)$, if $n>1$

(c) $\frac{5}{4}(1+i)$, if $n=1$ (d) 0

Comprehension Type

Consider the equation $x^4 - \lambda x^2 + 9 = 0$.

14. If the equation has four real and distinct roots, then λ lies in the interval

(a) $(-\infty, -6) \cup (6, \infty)$ (b) $(0, \infty)$

(c) $(6, \infty)$ (d) $(-\infty, -6)$

15. If the equation has no real root, then λ lies in the interval

(a) $(-\infty, 0)$ (b) $(-\infty, 6)$

(c) $(6, \infty)$ (d) $(0, \infty)$

Matrix Match Type

16. Match the following:

Column I		Column II	
P.	If $(3a - 2ib)(2 + i)^2 = 10(1 + i)$, then reciprocal of $(a-3b)$ is	1.	3
Q.	If $z + z^3 = 0$, then Re(z) is equal to	2.	2
R.	If α is a complex constant such that $\alpha z^2+z+\bar{\alpha}=0$ has a real root, then value of $4|\alpha + \bar{\alpha}|$ is	3.	4
S.	If $x, y \in R$ and $2x^2 + 6xy + 5y^2 = 1$, then maximum value of y^2 is	4.	0

	P	Q	R	S
(a)	1	4	3	2
(b)	2	1	3	4
(c)	3	1	4	2
(d)	3	4	1	2

Numerical Answer Type

17. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + 2z_2 + 3z_3| = 6$, then find the value of $|z_2z_3 + 8z_3z_1 + 27z_1z_2|$.

18. If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root and $a \neq 0$, then $\frac{a^3+b^3+c^3}{abc}=$ _____.

19. If $|z| = \max\{|z - 1|, |z + 1|\}$, then value of $|z+\bar{z}|$ is _____.

20. The value of a for which the sum of the squares of the roots of the equation $x^2-(a - 2)x - a - 1 = 0$ assume the least value is _____.

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SELF CHECK

No. of questions attempted ……

No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

CLASS-XII

TERM-I OBJECTIVE TYPE QUESTIONS*

Series 1 Relations and Functions

MCQs

1. If $f : R \to R$, $g : R \to R$ and $h : R \to R$ is such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of $[ho(gof)](x)$, if $x = \frac{\sqrt{\pi}}{2}$ will be

(a) 0 (b) 1 (c) −1 (d) 10

2. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. Then, R is

(a) reflexive only (b) symmetric only
(c) transitive only (d) none of these

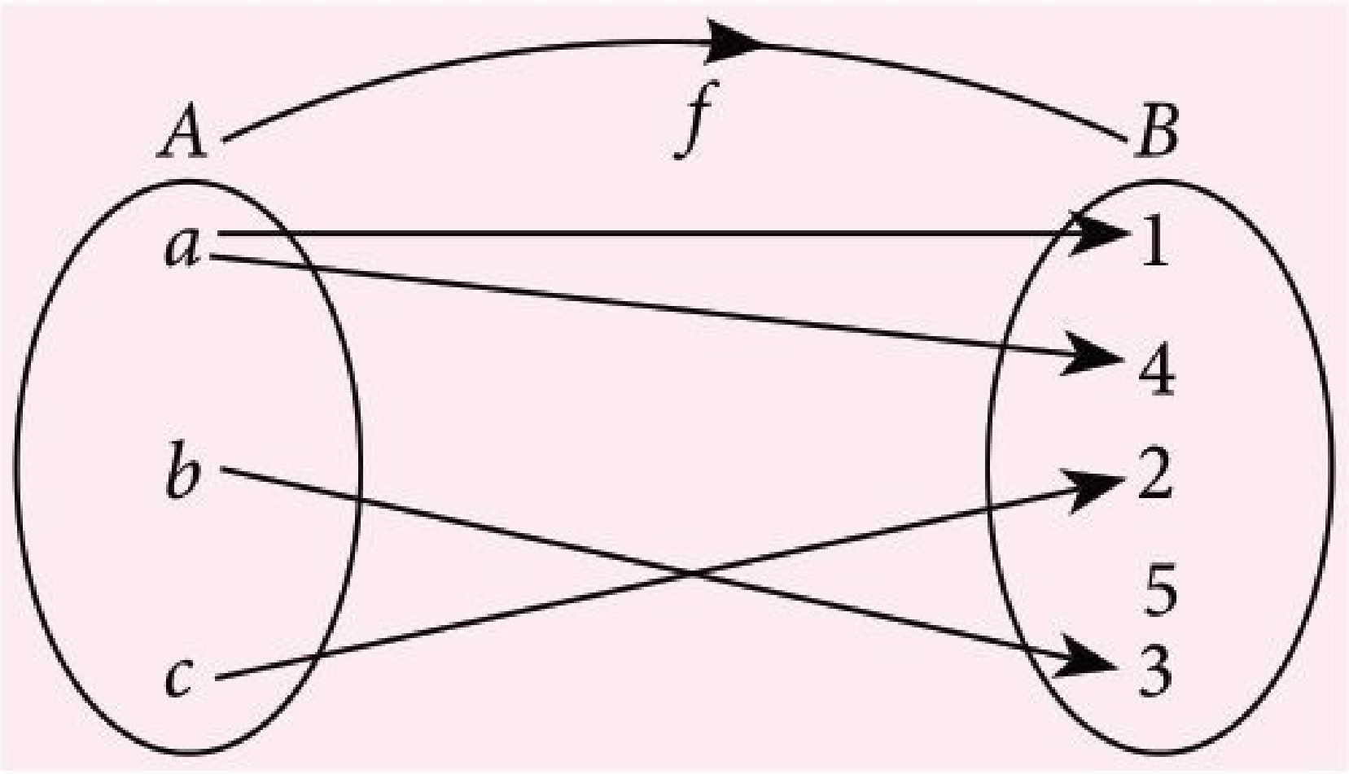
3. If $f : R \to R$ and $g : R \to R$ defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the value of x for which $f(g(x)) = 25$ is

(a) ±1 (b) ±2 (c) ±3 (d) ±4

4. The relation R is defined on the set of natural numbers as $\{(a, b): a = 2b\}$. Then, R^{-1} is given by

(a) $\{(2, 1), (4, 2), (6, 3),...\}$
(b) $\{(1, 2), (2, 4), (3, 6),...\}$
(c) R^{-1} is not defined
(d) none of these

5. The diagram given below shows that



(a) f is a function from A to B
(b) f is a one-one function from A to B
(c) f is an onto function from A to B
(d) f is not a function

6. If $f : R \to R$ defined by $f(x) = \frac{2x - 7}{4}$ is an invertible function, then find f^{-1}.

(a) $\frac{4x+5}{2}$ (b) $\frac{4x+7}{2}$
(c) $\frac{3x+2}{2}$ (d) $\frac{9x+3}{5}$

7. Find gof, if $f(x) = |x|$ and $g(x) = |5x - 2|$.

(a) $|5|x| - 2|$ (b) $|15x - 3|$
(c) $|5|x| - 3|$ (d) $|5x - 3|$

8. Find the interval in which the function $f(x) = 2\sin\left(2x - \frac{\pi}{6}\right) + 4$ is one-one.

(a) $\left[\frac{\pi}{2}, \frac{\pi}{3}\right]$ (b) $\left[\frac{-\pi}{6}, \frac{\pi}{4}\right]$
(c) $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$ (d) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

9. If $f(x) = x^2 + 3$ and $g(x) = \sqrt{x^2 - 2}$, then find $fog(x)$.

(a) $x + 1$ (b) $-x + 1$
(c) $-x^2 + 1$ (d) $x^2 + 1$

10. Set A has three elements and set B has four elements. Find the number of injections that can be defined from A to B.

(a) 22 (b) 24 (c) 26 (d) 28

***Chapterwise practice questions for CBSE Exam Term- I as per the pattern issued by CBSE.**

11. Let f be an invertible real function. Write $(f^{-1} of)\ (1) + (f^{-1} of)\ (2) + ... + (f^{-1} of)\ (100)$.

(a) 5010 (b) 2020
(c) 5050 (d) 1050

12. If $f(x) = x + 7$ and $g(x) = x - 7, x \in R$, then write $fog\ (7)$.

(a) 7 (b) 114 (c) 20 (d) 21

13. Let $f : R \rightarrow R$ be defined as $f(x) = x^2 + 1$. Find $f^{-1}\{10, 37\}$.

(a) $\{3, -3\}$ (b) $\{6, -6\}$
(c) $\{3, 6\}$ (d) $\{3, -3, 6, -6\}$

14. If $f(x)$ is an invertible function, then find the inverse of $f(x) = \dfrac{3x-2}{5}$.

(a) $5x + 2$ (b) $\dfrac{5x-2}{3}$
(c) $\dfrac{5x+2}{3}$ (d) $5x - 2$

15. Let $f : Z \rightarrow Z$ be defined by $f\ (x) = x + 2$. Find $g : Z \rightarrow Z$ such that $gof = I_Z$.

(a) $x - 2$ (b) $x + 2$ (c) $2x - 3$ (d) $2x + 3$

16. If f and g are two functions from R to R defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, then $fog(x)$ for $x < 0$ is

(a) $4x$ (b) $2x$ (c) 0 (d) $-4x$

17. Let $f : \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g : \{1, 2, 5\} \rightarrow \{1, 3\}$ given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

(a) $\{(1, 3), (3, 1), (4, 3)\}$
(b) $\{(3, 1), (1, 3), (3, 4)\}$
(c) $\{(1, 2), (2, 3), (3, 1)\}$
(d) $\{(1, 3), (3, 4), (4, 5)\}$

18. If $f : R \rightarrow R$ defined as $f(x) = \dfrac{x-1}{x-2}$ is an invertible function, then write $f^{-1}(x)$.

(a) $\dfrac{4x-1}{x-1}$ (b) $\dfrac{2x-1}{x-1}$
(c) $\dfrac{4x+7}{x-2}$ (d) $\dfrac{x-2}{x-1}$

19. Let f and g be two real functions given by $f(x) = 2x - 3$ and $g(x) = \dfrac{3+x}{2}$. Find fog.

(a) $\dfrac{x}{2}$ (b) x (c) $2x$ (d) $\dfrac{x}{3}$

20. The range of the function $f(x) = 3x^2 + 7x + 10$ is

(a) $[10, \infty)$ (b) $\left[\dfrac{71}{12}, \infty\right)$
(c) $[1, \infty)$ (d) none of these

21. If (1, 3), (2, 5) and (3, 3) are three elements of $A \times B$ and the total number of elements in $A \times B$ is 6, then the remaining elements of $A \times B$ are

(a) (1, 5), (2, 3), (3, 5)
(b) (5, 1), (3, 2), (5, 3)
(c) (1, 5), (2, 3), (5, 3)
(d) None of these

22. If $R = \{(x, y) : x, y \in I, x^2 + y^2 \leq 4\}$ is a relation in I, then domain of R is

(a) $\{0, 1, 2\}$ (b) $\{-2, -1, 0\}$
(c) $\{-2, -1, 0, 1, 2\}$ (d) None of these

23. The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is

(a) Reflexive but not symmetric
(b) Reflexive but not transitive
(c) Symmetric and transitive
(d) Neither symmetric nor transitive

24. Let $P = \{(x, y) : x^2 + y^2 = 1, x, y \in R\}$. Then, P is

(a) Reflexive (b) Symmetric
(c) Transitive (d) Anti-symmetric

25. Let L denote the set of all straight lines in XY plane. Let a relation R be defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Then, R is

(a) Reflexive only (b) Symmetric only
(c) Transitive only (d) An equivalence relation

26. Let $A = \{1, 2, 3, 4\}$ and R be a relation in A given by $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1)\}$. Then, R is

(a) Reflexive
(b) Transitive
(c) An equivalence relation
(d) None of these

27. Let R be a relation on the set N of natural numbers denoted by $nRm \Leftrightarrow n$ is a factor of m (*i.e.* $n \mid m$). Then, R is

(a) Reflexive and symmetric
(b) Transitive and symmetric
(c) Equivalence
(d) Reflexive, transitive but not symmetric

28. Let R be an equivalence relation on a finite set A having n elements. Then, the number of ordered pairs in R is

(a) Less than n
(b) Greater than or equal to n
(c) Less than or equal to n
(d) None of these

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29. The function $f: R \to R$, defined by $f(x) = 6^x + 6^{|x|}$ is
(a) One-one and onto (b) Many-one and onto
(c) One-one and into (d) Many-one and into

30. $f: N \to N$, where $f(x) = x - (-1)^x$, then f is
(a) One-one and into (b) Many-one and into
(c) One-one and onto (d) Many-one and onto

31. Let $X = \{0, 1, 2, 3\}$ and $Y = \{-1, 0, 1, 4, 9\}$ and a function $f: X \to Y$ defined by $y = x^2$, is
(a) one-one onto (b) one-one into
(c) many-one onto (d) many-one into

32. The mapping $f: N \to N$ given by $f(n) = 1 + n^2$, $n \in N$, where N is the set of natural numbers, is
(a) one-one and onto
(b) onto but not one-one
(c) one-one but not onto
(d) neither one-one nor onto

33. The function $f: R \to R$ given by $f(x) = x^3 - 1$ is
(a) a one-one function
(b) an onto function
(c) a bijection
(d) neither one-one nor onto

34. The smallest integer function $f(x) = \lceil x \rceil$ is
(a) One-one (b) Many-one
(c) Both (a) & (b) (d) None of these

35. The signum function, $f: R \to R$ is given by

$$f(x)=\begin{cases}1, & \text{if } x>0\\ 0, & \text{if } x=0\\ -1, & \text{if } x<0\end{cases} \quad \text{is}$$

(a) One-one (b) Onto
(c) Many-one (d) None of these

36. Let $f: R \to R$, defined by $f(x) = 3x^2 - 5$ and $g: R \to R$, defined by $g(x) = \dfrac{x}{x^2+1}$. Then gof is

(a) $\dfrac{3x^2-5}{9x^4-30x^2+26}$ (b) $\dfrac{3x^2+5}{9x^4-30x^2+26}$

(c) $\dfrac{3x^2+5}{9x^4+30x^2-26}$ (d) $\dfrac{3x^2-5}{9x^4+30x^2+26}$

37. The number of bijective functions from set A to itself when A contains 106 elements is
(a) 106 (b) $(106)^2$
(c) 106! (d) 2^{106}

38. If $f: R \to R$ and $g: R \to R$ defined by $f(x) = 2x - 3$ and $g(x) = x^3 + 5$, then the value of x for which $f(g(x)) = 9$, is
(a) 1 (b) 2 (c) 3 (d) 4

39. Let the functions f, g, h are defined from R to R such that $f(x) = x^2 - 1$, $g(x) = \sqrt{x^2+1}$ and $h(x) = x^2$, then $ho(fog)(x)$ is defined by
(a) x^2 (b) x^4
(c) 0 (d) None of these

40. If $f: R \to R$ is given by

$$f(x)=\begin{cases}-1, & \text{when } x \text{ is rational}\\ 1, & \text{when } x \text{ is irrational}\end{cases},$$

then $(fof)(1-\sqrt{3})$ is equal to
(a) 1 (b) -1 (c) $\sqrt{3}$ (d) 0

41. If $f: [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \dfrac{1}{x}$, then $f^{-1}(x)$ equals

(a) $\dfrac{x+\sqrt{x^2-4}}{2}$ (b) $\dfrac{x}{1+x^2}$

(c) $\dfrac{x-\sqrt{x^2-4}}{2}$ (d) $1+\sqrt{x^2-4}$

42. Given $f(x) = \log\left(\dfrac{1+x}{1-x}\right)$ and $g(x) = \dfrac{3x+x^3}{1+3x^2}$, then $fog(x)$ equals
(a) $-f(x)$ (b) $3f(x)$
(c) $[f(x)]^3$ (d) None of these

43. Let $f(x) = \dfrac{ax}{x+1}$, $x \neq -1$. Then value of the constant a such that $f(f(x)) = x$ is
(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) -1

44. If $f: R \to R$, $g: R \to R$ and $h: R \to R$ are such that $f(x) = x + 2$, $g(x) = 5x$ and $h(x) = 8x$, then the value of $(go(foh))(x)$, if $x = 1$ will be
(a) 50 (b) 45 (c) -50 (d) 48

45. Consider $f: R^+ \to (4, \infty)$ given by $f(x) = x^2 + 4$. If $f(x)$ is invertible, then find the inverse of $f(x)$, where R^+ is the set of all non-negative real numbers.
(a) $\sqrt{y-4}$ (b) $\sqrt{x+4}$
(c) $\sqrt{x-4}$ (d) $\sqrt{y+4}$

46. Consider the function f in $A = R - \left\{\dfrac{2}{3}\right\}$ defined as $f(x) = \dfrac{4x+3}{6x-4}$. Find f^{-1}.

(a) $\dfrac{3+4x}{6x-4}$ (b) $\dfrac{6x-4}{3+4x}$

(c) $\dfrac{3-4x}{6x-4}$ (d) $\dfrac{9+2x}{6x-4}$

47. Let $f : R \to R$, $g : R \to R$ be two functions such that $f(x) = 2x + 3$, $g(x) = x^3 + 5$. The function $(fog)^{-1}(x)$ is equal to

(a) $\left(\frac{x+13}{2}\right)^{1/3}$ (b) $\left(x-\frac{13}{2}\right)^{1/3}$

(c) $\left(\frac{x-13}{7}\right)^{1/3}$ (d) $\left(\frac{x-13}{2}\right)^{1/3}$

48. If N be the set of all natural numbers, consider $f : N \to N$ such that $f(x) = 2x$, $\forall\, x \in N$, then f is

(a) one-one onto (b) one-one into

(c) many-one onto (d) None of these

49. If $f(x) = 1 - \frac{1}{x}$, then $f\left(f\left(\frac{1}{x}\right)\right)$ is

(a) $\frac{1}{x}$ (b) $\frac{1}{1+x}$

(c) $\frac{x}{x-1}$ (d) $\frac{1}{x-1}$

50. If $f(x) = \frac{3x+2}{5x-3}$, then $(fof)(x)$ is

(a) x (b) $-x$

(c) $f(x)$ (d) $-f(x)$

CASE BASED

Case-I : Arjun was given a project by his class teacher to form set of school going children in two different societies including girls as well as boys. He form the set as follows:

$A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$, where a_i's and b_i's are the school going children of first and second society respectively. Explore these sets for various types of relations and functions and answer the following questions.

51. Number of possible reflexive relations defined on set A is

(a) 0 (b) 1

(c) 2^5 (d) 2^{20}

52. Let $R : A \to A$ such that $R = \{(x, y) : x$ and y are students of same gender$\}$. Then relation R is

(a) Reflexive only

(b) Reflexive and symmetric but not transitive

(c) Reflexive and transitive but not symmetric

(d) An equivalence relation

53. Number of symmetric relations defined on set B is

(a) 2^5 (b) 2^{10}

(c) 2^{15} (d) 2^{20}

54. Let $R : A \to B$ such that $R = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$, then R is

(a) one-one but not onto

(b) onto but not one-one

(c) neither one-one nor onto

(d) not a function

55. Number of onto functions defined from set A to set B is

(a) 200 (b) 240

(c) 250 (d) 1005

Case-II : Anjali visited the park with her parents, where she observed a swing, which attracted her. She found that the movement of swing traced the path of a parabola as given by $y = x^2$. As, Anjali is Class XII student, she explored the shape and try to find the answer of following questions.

56. $f : R \to R$ defined by $f(x) = x^2$, is

(a) one-one

(b) onto

(c) neither one-one nor onto

(d) both one-one and onto

57. $f : N \to N$ defined by $f(x) = x^2$ is

(a) one-one only

(b) onto only

(c) both one-one and onto

(d) neither one-one nor onto

58. Range of the function $f(x) = x^2$ is

(a) $(-\infty, 0)$ (b) $(0, \infty)$

(c) $[0, \infty)$ (d) $(-\infty, 0]$

59. Domain of function $f(x) = x^2$ is

(a) $(-\infty, \infty)$ (b) $(0, \infty)$

(c) $(-\infty, 0)$ (d) $[0, \infty)$

60. $f:\{1, 2, 3, ...\} \to \{1, 4, 9,...\}$ defined by $f(x) = x^2$ is
(a) onto
(b) one-one
(c) both one-one and onto
(d) neither one-one nor onto

ASSERTION & REASON

Directions (Q.No. 61-70) : In the following questions, a statement of assertion (Statement-I) is followed by a statement of reason (Statement-II). Mark the correct choice as :

(a) If both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.
(b) If both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.
(c) If Statement-I is true but Statement-II is false.
(d) If Statement-I is false but Statement-II is true.

61. **Statement-I :** If $f : R \to R$ given by $f(x) = 2x + 3$, then $f^{-1}(x) = \dfrac{x+3}{2}$.
Statement-II : If $y = f(x)$, then $x = f^{-1}(y)$.

62. **Statement-I :** Let $f : N \to y$ given by $f(x) = 4x^2 + 12x + 15$ and y = range (f), then f is invertible.
Statement-II : If $f : A \to B$ is bijective, then f is invertible function.

63. **Statement-I :** Let N be the set of all natural numbers and R be a relation in N defined by $R = \{(a, b) : a \text{ is a multiple of } b\}$. Then R is reflexive and transitive but not symmetric.
Statement-II : A relation R is an equivalence relation if R is reflexive and symmetric only.

64. **Statement-I :** For $f(x) = \{(3, 4), (1, -2), (5, -1), (0, 2)\}$, inverse relation is also a function.
Statement-II : An inverse relation will be a function only if the original function is one-one.

65. **Statement-I :** Let $A = \{-1, 1, 2, 3\}$ and $B = \{1, 4, 9\}$, where $f : A \to B$ given by $f(x) = x^2$, then f is a many-one function.
Statement-II : If $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$, for every $x_1, x_2 \in$ domain, then f is one-one or else many-one.

66. **Statement-I :** If $f : R \to R$ and $g : R \to R$ be two mappings such that $f(x) = \sin x$ and $g(x) = x^2$, then $fog \neq gof$.
Statement-II : $(fog)x = f(x)g(x) = (gof)x$.

67. **Statement-I :** $f : Z \to Z$ given by $f(x) = x^2 + 1\ \forall\ x \in Z$ is not one-one.
Statement-II : If A and B are two sets having m and n elements respectively such that $m \leq n$, then total number of one-one functions from A to B is ${}^nC_m.m!$.

68. **Statement-I :** Let A be a finite set. If $f : A \to A$ is a one-one function, then f is onto also.
Statement-II : Let A be a finite set. If $f : A \to A$ is an onto function, then f is not one-one.

69. **Statement-I :** A relation R on a set A is called symmetric, if $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R\ \forall\ a_1, a_2 \in A$.
Statement-II : A relation on a set A is called reflexive relation, if each element of A is related to itself.

70. **Statement-I :** If $f(x) = \sin^2 x + \sin^2\left(x+\dfrac{\pi}{3}\right) + \cos x \cdot \cos\left(x+\dfrac{\pi}{3}\right)$ and $g\left(\dfrac{5}{4}\right) = 1$, then $gof(x)$ is not a constant function.
Statement-II : Let $f : A \to B$ and $g : B \to C$ be two functions of non empty sets A, B and C, then $gof : A \to C$ is called composition of f and g defined as $gof(x) = g\{f(x)\}\ \forall\ x \in A$.

SOLUTIONS

1. **(a) :** $[ho(gof)](x) = ho[g(f(x))]$

$= h[g(x^2)] = h[\tan x^2] = h\left[\tan\dfrac{\pi}{4}\right] = h[1] = \log 1 = 0$

2. **(b) :** Given, $\alpha R \beta \Leftrightarrow \alpha \perp \beta$

$\therefore\ \alpha \perp \beta \Leftrightarrow \beta \perp \alpha \Rightarrow \beta R \alpha$

Hence, R is symmetric.

3. **(b) :** $f(g(x)) = f(x^2 + 7) = 2(x^2 + 7) + 3 = 25$ (given)
$\Rightarrow\ 2x^2 + 17 = 25 \Rightarrow 2x^2 = 8\ \Rightarrow\ x = \pm 2$

4. **(b) :** $R = \{(2, 1), (4, 2), (6, 3),...\}$
So, $R^{-1} = \{(1, 2), (2, 4), (3, 6), ...\}$

5. **(d) :** As $f(a)$ is not unique, thus f is not a function.

6. **(b) :** Let $y = f(x) = \dfrac{2x-7}{4}\ \Rightarrow\ x = \dfrac{4y+7}{2}$

$\therefore\ f^{-1}(y) = \dfrac{4y+7}{2}\ \Rightarrow\ f^{-1}(x) = \dfrac{4x+7}{2}$

7. **(a) :** Given $f(x) = |x|$ and $g(x) = |5x - 2|$
Now $gof(x) = g(f(x)) = g(|x|) = |5|x| - 2|$

8. **(c) :** We have $f(x) = 2\sin\left(2x - \dfrac{\pi}{6}\right) + 4$

Since, $\sin x$ is one-one in $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$$\therefore \quad -\frac{\pi}{2} \le 2x - \frac{\pi}{6} \le \frac{\pi}{2} \Rightarrow x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$$

$\therefore$ The required interval is $\left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$.

9. (d) : $fog(x) = f(g(x)) = f(\sqrt{x^2-2})$

$= (\sqrt{x^2-2})^2 + 3 = x^2 - 2 + 3 = x^2 + 1$

10. (b) : Since 3 < 4, injective functions from A to B are defined and the total number of such functions is $^4P_3 = \frac{4!}{(4-3)!} = 24$

11. (c) : Since, f is an invertible function, so $(f^{-1}of)(x) = x$

$\therefore \quad (f^{-1}of)(1) + (f^{-1}of)(2) + \ldots + (f^{-1}of)(100)$

$= 1 + 2 + \ldots + 100$

$= 100 \times \frac{(100+1)}{2} = 5050$

12. (a) : $fog(x) = f(g(x)) = f(x-7) = x - 7 + 7 = x$

$\Rightarrow \; fog(7) = 7$

13. (d) : $f^{-1}\{10, 37\} = \{x \in R : f(x) = 10 \text{ or } f(x) = 37\}$

$= \{x \in R : x^2 + 1 = 10 \text{ or } x^2 + 1 = 37\}$

$= \{x \in R : x^2 = 9 \text{ or } x^2 = 36\} = \{3, -3, 6, -6\}$

14. (c) : Let $y = f(x) = \frac{3x-2}{5}$

$\Rightarrow \; 5y = 3x - 2 \; \Rightarrow \; x = \frac{5y+2}{3} \; \Rightarrow \; f^{-1}(y) = \frac{5y+2}{3}$

$\therefore \quad f^{-1}(x) = \frac{5x+2}{3}$

15. (a) : Given $f(x) = x + 2$

Now, $gof = I_Z \; \Rightarrow \; (gof)(x) = I_Z(x)$, for all $x \in Z$

$\Rightarrow \; g(f(x)) = x$, for all $x \in Z$

$\Rightarrow \; g(x+2) = x$, for all $x \in Z$

$\Rightarrow \; g(x) = x - 2$, for all $x \in Z$ [Replace x by $x - 2$]

Hence the required function $g : Z \to Z$ is given by $g(x) = x - 2$, for all $x \in Z$.

16. (d) : Here, $f, g : R \to R$ such that

$$f(x) = |x| + x = \begin{cases} 2x, & \text{if } x \ge 0 \\ 0, & \text{if } x < 0 \end{cases}$$

and $g(x) = |x| - x = \begin{cases} 0, & \text{if } x \ge 0 \\ -2x, & \text{if } x < 0 \end{cases}$

$$\therefore \quad (fog)(x) = f(g(x)) = \begin{cases} f(0), & \text{if } x \ge 0 \\ f(-2x), & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} 0, & \text{if } x \ge 0 \\ 2(-2x), & \text{if } x < 0 \end{cases} = \begin{cases} 0, & \text{if } x \ge 0 \\ -4x, & \text{if } x < 0 \end{cases}$$

17. (a) : Here, $f = \{(1, 2), (3, 5), (4, 1)\}$

$\Leftrightarrow \; f(1) = 2; f(3) = 5; f(4) = 1$ ...(i)

$g = \{(1, 3), (2, 3), (5, 1)\}$

$\Leftrightarrow \; g(1) = 3; g(2) = 3, g(5) = 1$ ...(ii)

Now, $gof : \{1, 3, 4\} \to \{1, 3\}$

From (i) and (ii), we get

$(gof)(1) = g(f(1)) = g(2) = 3$

$(gof)(3) = g(f(3)) = g(5) = 1$

$(gof)(4) = g(f(4)) = g(1) = 3$

$\therefore \quad gof = \{(1, 3), (3, 1), (4, 3)\}$.

18. (b) : Let $y = f(x) = \frac{x-1}{x-2} \Rightarrow xy - 2y = x - 1$

$\Rightarrow \; x(y-1) = 2y - 1$

$\Rightarrow \; x = \frac{2y-1}{y-1} \; \Rightarrow f^{-1}(y) = \frac{2y-1}{y-1}$

$\therefore \quad f^{-1}(x) = \frac{2x-1}{x-1}$

19. (b) : $fog(x) = f(g(x)) = f\left(\frac{3+x}{2}\right) = 2\left(\frac{3+x}{2}\right) - 3$

$= 3 + x - 3 = x$

20. (b) : If $y = f(x)$, then $y = 3x^2 + 7x + 10$

$\Rightarrow \; 3x^2 + 7x + 10 - y = 0$

Since x is real, $(7)^2 - 4 \cdot 3(10 - y) \ge 0 \Rightarrow y \ge \frac{71}{12}$

$\Rightarrow \quad y \in \left[\frac{71}{12}, \infty\right)$

21. (a) : $A = \{1, 2, 3\}$, $B = \{3, 5\}$

$\therefore \quad A \times B = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$

22. (c) : Given, $R = \{(x, y) : x, y \in I, x^2 + y^2 \le 4\} = \{(0,0), (0, -1), (0, 1), (0, -2), (0, 2), (-1, 0), (-1, 1), (-1, -1), (1, 0), (1, -1), (1, 1), (2, 0), (-2, 0)\}$

$\therefore$ Domain of $R = \{x : (x, y) \in R\} = \{-2, -1, 0, 1, 2\}$

23. (a) : Reflexive : $(1, 1), (2, 2), (3, 3) \in R$

$\therefore$ R is reflexive

Symmetric : $(1, 2) \in R$ but $(2, 1) \notin R$

$\therefore$ R is not symmetric.

Transitive : $(1, 2) \in R$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$, R is transitive.

24. (b) : The relation is not reflexive and transitive but it is symmetric, because $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$

25. (d) : Reflexive : As, L_1 is parallel to itself *i.e.*, $(L_1, L_1) \in R$

$\therefore$ R is reflexive.

Symmetric : Let $(L_1, L_2) \in R \Rightarrow (L_2, L_1) \in R$

$\therefore$ R is symmetric

Transitive : Let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow \; (L_1, L_3) \in R$

$\therefore$ R is transitive

Hence, R is an equivalence relation.

26. (a) : Since, (1, 1), (2, 2), (3, 3), (4, 4) $\in R$.
$\therefore$ R is reflexive.

27. (d) : Reflexive : $n \mid n$ for all $n \in N \Rightarrow R$ is reflexive.
Symmetric : $2 \mid 6$ but $6 \mid 2 \Rightarrow R$ is not symmetric.
Transitive : Let nRm and mRp
$\Rightarrow n \mid m$ and $m \mid p$
$\Rightarrow$ $n \mid p \Rightarrow nRp$. So, R is transitive.

28. (b) : As R is an equivalence relation on set A. Hence, R has atleast n ordered pairs.

29. (d) : $6^x > 0$, $6^{|x|} > 0$ for all $x \in R$.
So, f is into. For different values of x, 6^x and $6^{|x|}$ are different positive numbers. Clearly, f is many-one.

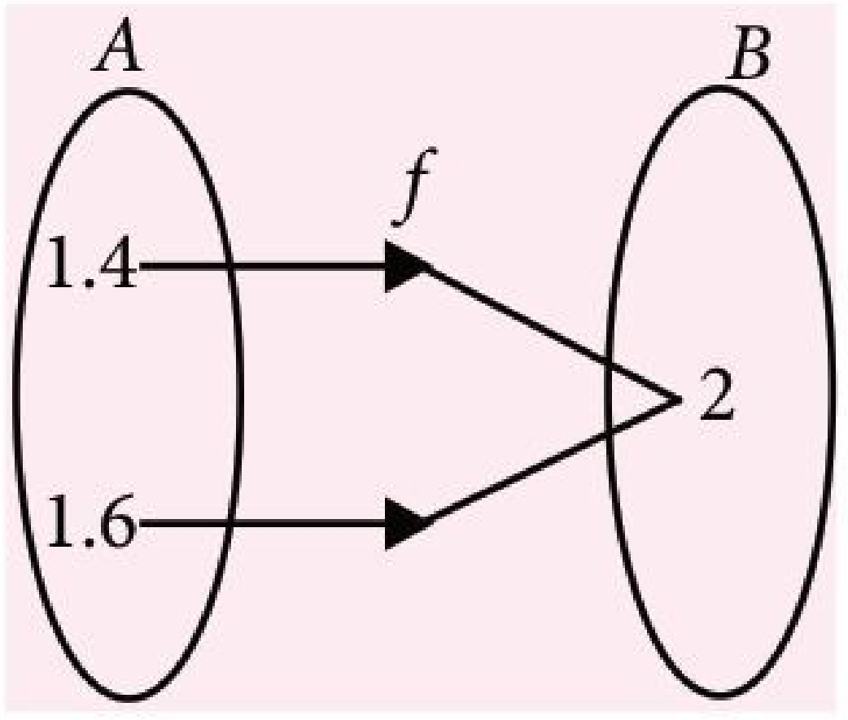
30. (c) : $f(x)=\begin{cases} x-1, & x \text{ is even} \\ x+1, & x \text{ is odd} \end{cases}$,
which is clearly one-one and onto.

31. (b) : $y(0) = 0$, $y(1) = 1$, $y(2) = 4$, $y(3) = 9$. No two different values of x (where $x \in X$) gives same image. Also -1 is element of set Y, which does not have its pre-image in set X. So, function is one-one into.

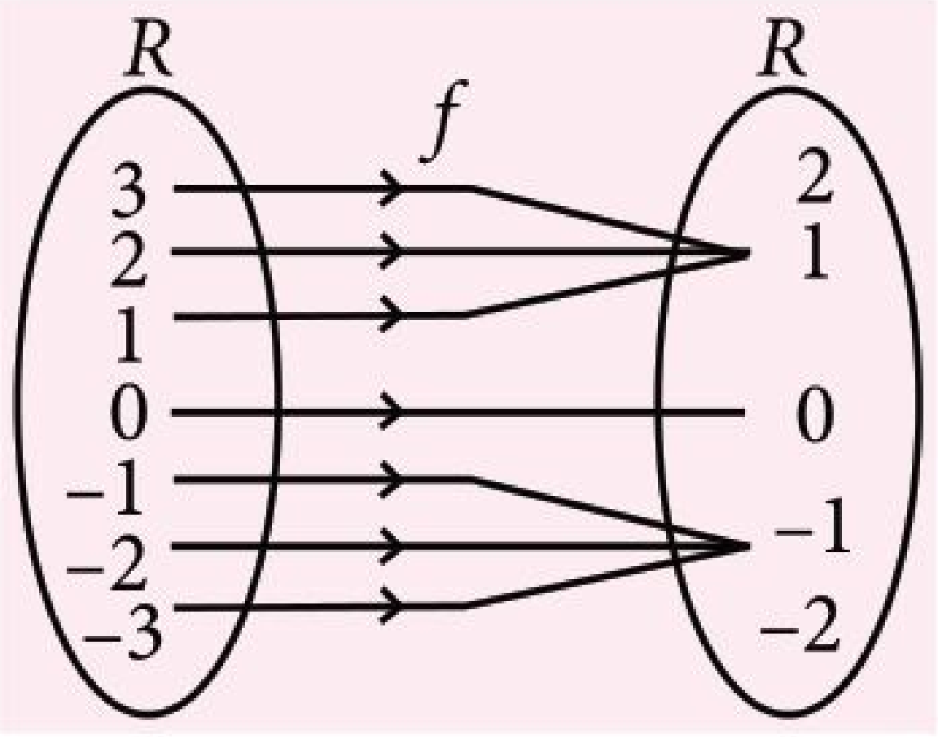
32. (c) : Since, $f(n) = 1 + n^2$
For one-one, $1+n_1^2 = 1+n_2^2$
$\Rightarrow$ $n_1^2 - n_2^2 = 0 \Rightarrow n_1 = n_2$ $\quad (\because n_1 + n_2 \neq 0)$
$\therefore$ $f(n)$ is one-one.
Clearly, $f(n)$ is not onto.

33. (c) : Given, $f(x) = x^3 - 1$. Let $x_1, x_2 \in R$.
Now, $f(x_1) = f(x_2) \Rightarrow x_1^3 - 1 = x_2^3 - 1 \Rightarrow x_1 = x_2$
$\therefore$ $f(x)$ is one-one. Also, it is onto as range of $f(x)$ is R. Hence, it is a bijection.

34. (b) : We have, $\lceil 1.4 \rceil = \lceil 1.6 \rceil = 2$
Here, two elements in A, 1.4 and 1.6 have the same image *i.e.*, 2 in B. Thus, $f(x) = \lceil x \rceil$ is a many-one function.



35. (c) : We have, $f(3) = f(2) = f(1) = 1$; $f(0) = 0$
$f(-1) = f(-2) = f(-3) = -1$
Hence, signum function is many-one function.



36. (a) : We have, $f(x) = 3x^2 - 5$, $g(x) = \dfrac{x}{x^2+1}$
$\therefore$ $gof(x) = g(f(x)) = g(3x^2 - 5)$

$$=\frac{3x^2-5}{(3x^2-5)^2+1}=\frac{3x^2-5}{9x^4-30x^2+26}$$

37. (c) : The total number of bijections from a set containing n elements to itself is $n!$.
Hence, required number = (106)!

38. (a) : $f(g(x)) = f(x^3 + 5) = 2(x^3 + 5) - 3 = 9$ (given)
$\Rightarrow$ $2x^3 + 7 = 9$ $\Rightarrow$ $x = 1$

39. (b) : $ho(fog)(x) = hof(\sqrt{x^2+1})$
$= h\{(\sqrt{x^2+1})^2 - 1\} = h\{x^2\} = x^4$

40. (b) : $(fof)(1-\sqrt{3}) = f[f(1-\sqrt{3})] = f(1) = -1$

41. (a) : Let $y = f(x) = x + \dfrac{1}{x}$ $\Rightarrow$ $x^2 - xy + 1 = 0$

$$\Rightarrow\ x=\frac{y\pm\sqrt{y^2-4}}{2}$$

$\because$ $y \in [2, \infty)$ $\Rightarrow$ $x \in [1, \infty)$

Hence, $f^{-1}(x)=\dfrac{x+\sqrt{x^2-4}}{2}$

42. (b) : $f[g(x)] = f\left[\dfrac{3x+x^3}{1+3x^2}\right] = \log\left\{\dfrac{1+\dfrac{3x+x^3}{1+3x^2}}{1-\dfrac{3x+x^3}{1+3x^2}}\right\}$

$$\Rightarrow\ f(g(x))=\log\left(\frac{1+x}{1-x}\right)^3=3\log\left(\frac{1+x}{1-x}\right)=3f(x)$$

43. (d) : $f(x) = \dfrac{ax}{x+1} \Rightarrow f(f(x)) = \dfrac{af(x)}{f(x)+1} = \dfrac{a\left(\dfrac{ax}{x+1}\right)}{\dfrac{ax}{x+1}+1}$

$= \dfrac{a^2x}{(a+1)x+1} = x$ if $a = -1$.

44. (a) : $(go(foh))(x) = gof(h(x)) = gof(8x)$
$= g(8x + 2) = 5(8x + 2)$
$\therefore$ $(go(foh))(1) = 5(8(1) + 2) = 50$

45. (c) : Given that $f(x) = x^2 + 4$
Let $y = x^2 + 4$ $\Rightarrow$ $x^2 = y - 4$ $\Rightarrow$ $x = \sqrt{y-4}$
$\therefore$ $f^{-1}(y) = \sqrt{y-4}$ $\Rightarrow$ $f^{-1}(x) = \sqrt{x-4}$

46. (a) : Let $y = f(x) = \dfrac{4x+3}{6x-4}$ $\Rightarrow$ $6xy - 4y = 4x + 3$

$\Rightarrow$ $x = \dfrac{3+4y}{6y-4}$ $\therefore$ $f^{-1}(x) = \dfrac{3+4x}{6x-4}$

47. (d) : $(fog)x = f(x^3 + 5) = 2(x^3 + 5) + 3 = 2x^3 + 13$

Let $y = (fog)x = 2x^3 + 13$ $\Rightarrow$ $x=\left(\dfrac{y-13}{2}\right)^{1/3}$

$$\Rightarrow\ (fog)^{-1}(x)=\left(\frac{x-13}{2}\right)^{1/3}$$

48. (b) : Let $x_1, x_2 \in N$, then $f(x_1) = f(x_2)$
$\Rightarrow$ $x_1 = x_2$. So, $f(x)$ is one-one.
Let $y = 2x \Rightarrow x = \frac{y}{2} \notin N$. Thus, f is into.
Hence, $f(x)$ is one-one into.

49. (c) : $f\left(f\left(\frac{1}{x}\right)\right) = f\left(1 - \frac{1}{1/x}\right)$

$$= f(1-x) = 1 - \frac{1}{1-x} = \frac{x}{x-1}$$

50. (a) : $(fof)(x) = f(f(x)) = f\left(\frac{3x+2}{5x-3}\right)$

$$= \frac{3\left(\frac{3x+2}{5x-3}\right)+2}{5\left(\frac{3x+2}{5x-3}\right)-3} = \frac{9x+6+10x-6}{15x+10-15x+9} = \frac{19x}{19} = x$$

51. (d): Since, number of reflexive relations defined on a set of n elements $= 2^{n(n-1)}$
$\therefore$ Number of reflexive relations defined on a set A having 5 elements $= 2^{5(5-1)} = 2^{20}$

52. (d) : As $(x, x) \in R \ \forall \ x \in A$; x is either boy or girl
$\Rightarrow$ R is reflexive.
Let $(x, y) \in R$; x, y are of same gender
$\Rightarrow$ y, x are of same gender
$\Rightarrow$ $(y, x) \in R$
$\Rightarrow$ R is symmetric.
Let $(x, y) \in R$ and $(y, z) \in R$
$\Rightarrow$ x, y are of same gender and y, z are of same gender
$\Rightarrow$ $(x, z) \in R$
$\Rightarrow$ R is transitive.
Hence, R is an equivalence relation.

53. (b) : $\because$ Number of symmetric relations defined on set of n elements $= 2^{\frac{n(n+1)}{2}}$
$\therefore$ Number of symmetric relations defined on set of 4 elements $= 2^{\frac{4(4+1)}{2}} = 2^{10}$

54. (d) : For the element $a_1 \in A$, we have different image under R, *i.e.*, $(a_1, b_1), (a_1, b_2) \in R$
$\therefore$ R is not a function.

55. (b) : $\because$ Number of onto functions defined from set A to set B

$$= \sum_{r=0}^{4} (-1)^r \, {}^4C_r (4-r)^5$$

[$\because$ Number of onto functions from set A to set B having m and n elements respectively $(m \geq n)$, is $\sum_{r=0}^{n} (-1)^r \, {}^nC_r (n-r)^m$].

$$= (-1)^0\,{}^4C_0(4-0)^5 + (-1)^1\,{}^4C_1(4-1)^5 + (-1)^2\,{}^4C_2(4-2)^5 + (-1)^3\,{}^4C_3(4-3)^5 + (-1)^4\,{}^4C_4(4-4)^5$$

$$= 4^5 - 4(3)^5 + 6(2)^5 - 4(1) + 0 = 1024 - 972 + 192 - 4 = 240$$

56. (c) : We have, $f(x) = x^2, x \in R$
For $x_1, x_2 \in R$,
$f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2$
$\Rightarrow$ $x_1 = x_2$ or $x_1 = -x_2$
$\therefore$ $f(x)$ is not one-one
Also $f(x) = y, y \in R$
$\Rightarrow$ $x^2 = y \Rightarrow x = \pm\sqrt{y}$
As y is real number, so can be negative, which will make x non-real.
$\therefore$ $f(x)$ is not onto.

57. (a) : $f(x) > 0 \ \forall \ x \in N$
$\because$ Every distinct element has its distinct image.
$\therefore$ f is one-one.
Let $f(x) = y, y \in N \Rightarrow x^2 = y \Rightarrow x = \pm\sqrt{y} \notin N$ for non perfect square values of y.
$\therefore$ $f(x)$ is not onto.

58. (c) : Since, $x = \pm\sqrt{y}$
For x to be defined, y must be greater than or equal to 0.
$\therefore$ Range $(f) = [0, \infty)$

59. (a) : $f(x) = x^2$ is defined for all $x \in R$
$\therefore$ Dom $(f) = R$, *i.e.*, $(-\infty, \infty)$

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60. (c) : Let $X = \{1, 2, 3,...\}$ and $Y = \{1, 4, 9,...\}$
Every distinct element in X has distinct image in Y.
$\therefore$ f is one-one.
Also $f(1) = 1, f(2) = 4, f(3) = 9$, ...
For every element of Y there exist pre image in X.
$\therefore$ f is an onto function.

61. (d) : Clearly, Statement-II is true.
Let $y = f(x) \Rightarrow y = 2x + 3$

$$\Rightarrow \quad x = \frac{y-3}{2} \Rightarrow f^{-1}(y) = \frac{y-3}{2} \Rightarrow f^{-1}(x) = \frac{x-3}{2}$$

$\therefore$ Statement-I is false.

62. (a) : Clearly, Statement-II is true.
For one-one : Let $x_1, x_2 \in N$ and consider $f(x_1) = f(x_2)$

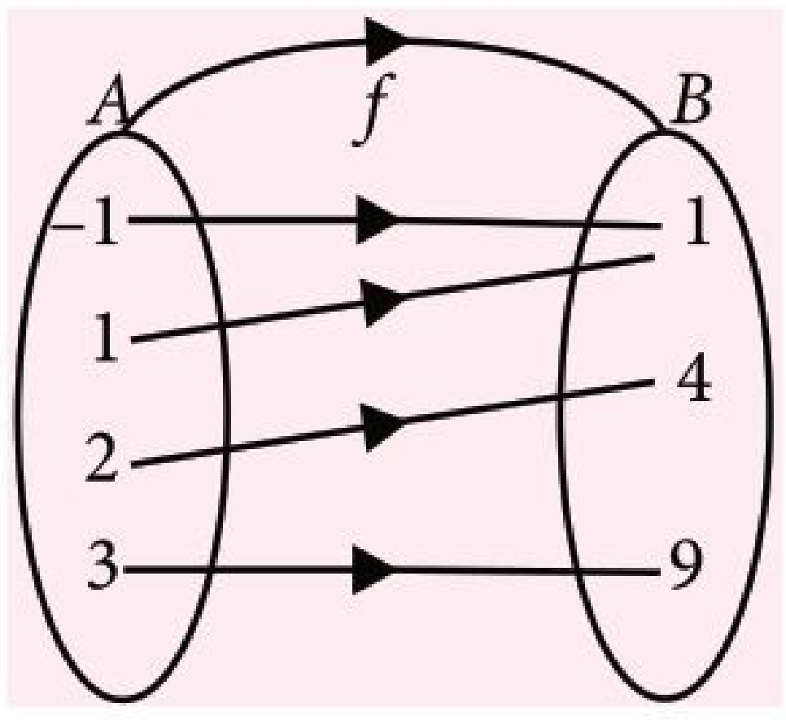
$$\Rightarrow \quad 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$$
$$\Rightarrow \quad 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0$$
$$\Rightarrow \quad (x_1 - x_2)(x_1 + x_2 + 3) = 0$$
$$\Rightarrow \quad x_1 - x_2 = 0 \qquad [\because x_1 + x_2 + 3 \neq 0]$$
$$\Rightarrow \quad x_1 = x_2 \quad \therefore f \text{ is one-one.}$$

For onto : Range (f) = y. So, f is onto.
Thus, f is one-one and onto *i.e.*, bijective.
$\therefore$ f is invertible.
$\therefore$ Both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.

63. (c) : Clearly, Statement-II is false, as we know that relation R is said to be an equivalence relation if R is reflexive, symmetric and transitive.
Now, Reflexivity : Let $a \in N$, then
a is a multiple of $a \Rightarrow (a, a) \in R$
$\therefore$ R is reflexive.
Symmetry : Let $(a, b) \in R$ *i.e.*, a is a multiple of b but $(b, a) \notin R$ *i.e.*, b is not a multiple of a.
$\therefore$ R is not symmetric.
Transitivity : Let $a, b, c \in N$ such that $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.
$\therefore$ R is transitive.
$\therefore$ Statement-I is true.

64. (a) : Clearly, Statement-II is true.
Here, f is one-one function as in ordered pair, values of x and y are used only once.
$\therefore$ $f^{-1}(x) = \{(4, 3), (-2, 1), (-1, 5), (2, 0)\}$ is also a function.
$\therefore$ Both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.

65. (a) : Here $f(-1) = 1, f(1) = 1, f(2) = 4, f(3) = 9$
Two elements 1 and -1 have the same image $1 \in B$.
So, f is a many-one function.



$\therefore$ Both Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.

66. (c) : Clearly, Statement-II is false.
Since, $(fog)x = f\{g(x)\} = f(x^2) = \sin x^2$ and
$(gof)x = g\{f(x)\} = g(\sin x) = \sin^2 x \Rightarrow fog \neq gof$.
$\therefore$ Statement - I is true.

67. (b) : Clearly, Statement-II is true.
Let $x, y \in Z$ be two arbitrary elements such that $f(x) = f(y) \Rightarrow x^2 + 1 = y^2 + 1 \Rightarrow x^2 = y^2 \Rightarrow x = \pm y$
Since, $f(x) = f(y)$ does not provide the unique solution $x = y$.
So, f is not one-one function.
$\therefore$ Statement-I is true.
$\therefore$ Both Statement-I and Statement-II are true but Statement-II is not the correct explanation of Statement-I.

68. (c) : Let $A = \{a_1, a_2, a_3,...,a_n\}$
Since, $f : A \to A$ is one-one function.
$\therefore$ $f(a_1), f(a_2), ..., f(a_n)$ are distinct element of set A.
But A has only n elements.
$\therefore$ $A = \{f(a_1), f(a_2), ..., f(a_n)\}$ *i.e.*, Co-domain = Range
Here, $f : A \to A$ is onto.
$\therefore$ Statement-I is true.
Let $A = \{a_1, a_2,...., a_n\}$.
Since, $f : A \to A$ is an onto function.
$\therefore$ Range of $f = A \Rightarrow \{f(a_1), f(a_2),...,f(a_n)\} = A$.
But A is finite set consisting of n elements.
$\therefore$ $f(a_1), f(a_2),..., f(a_n)$ are distinct elements of A.
Hence, $f : A \to A$ is one-one.
$\therefore$ Statement-I is true but Statement-II is false.

69. (b)

70. (d) : Clearly, Statement-II is true.
Now, $gof(x) = g(f(x))$

$$= g\left[\sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cdot \cos\left(x + \frac{\pi}{3}\right)\right]$$

$$= g\left\{\sin^2 x + \left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)^2 + \cos x\left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right)\right\}$$

$$= g\left(\frac{5}{4}\sin^2 x + \frac{5}{4}\cos^2 x\right) = g(5/4) = 1$$

Hence, $gof(x)$ is a constant function.
$\therefore$ Statement-I is false.

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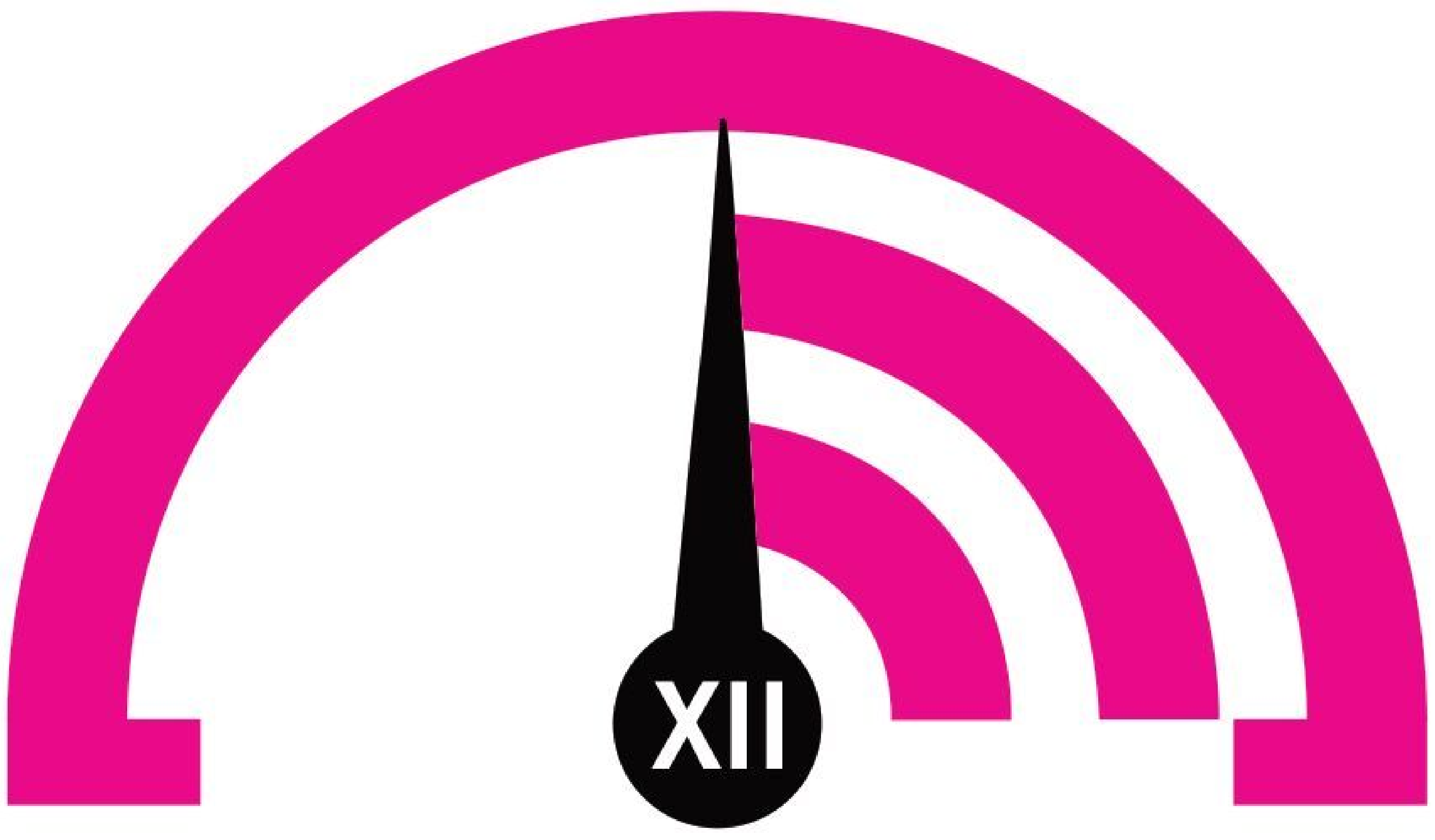


This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 2 : Inverse Trigonometric Functions

Time Taken : 60 Min.

Only One Option Correct Type

1. If $x_1 = 2\tan^{-1}\left(\frac{1+x}{1-x}\right), x_2 = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, where $x \in (0, 1)$, then $x_1 + x_2$ is equal to

(a) 0 (b) 2π

(c) π (d) none of these

2. If x_1, x_2, x_3 and x_4 are the roots of the equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x \cos\beta - \sin\beta = 0$, then $\tan^{-1}x_1 + \tan^{-1}x_2 + \tan^{-1}x_3 + \tan^{-1}x_4$ is equal to

(a) β (b) $\frac{\pi}{2}-\beta$

(c) $\pi - \beta$ (d) $-\beta$

3. If $[\sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1}x] = 1$, where $[\cdot]$ denotes the greatest integer function, then x belongs to the interval

(a) [tan sin cos 1, tan sin cos sin 1]

(b) (tan sin cos 1, tan sin cos sin 1)

(c) $[-1, 1]$

(d) [sin cos tan 1, sin cos sin tan 1]

4. The smallest and the largest values of $\tan^{-1}\left(\frac{1-x}{1+x}\right), 0 \le x \le 1$ are

(a) $0, \pi$ (b) $0, \frac{\pi}{4}$

(c) $-\frac{\pi}{4}, \frac{\pi}{4}$ (d) $\frac{\pi}{4}, \frac{\pi}{2}$

5. Let $\cos^{-1}(x) + \cos^{-1}(2x) + \cos^{-1}(3x) = \pi$. If x satisfies the cubic $ax^3 + bx^2 + cx - 1 = 0$, then $(a + b + c)$ is equal to

(a) 24 (b) 25

(c) 26 (d) 27

6. If $\alpha = 2\tan^{-1}(\sqrt{2}-1), \beta = 3\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\left(-\frac{1}{2}\right)$ and $\gamma = \cos^{-1}\frac{1}{3}$. Then,

(a) $\alpha < \beta < \gamma$ (b) $\alpha < \gamma < \beta$

(c) $\beta < \gamma < \alpha$ (d) $\gamma < \beta < \alpha$

One or More than One Option(s) Correct Type

7. The value of $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 0

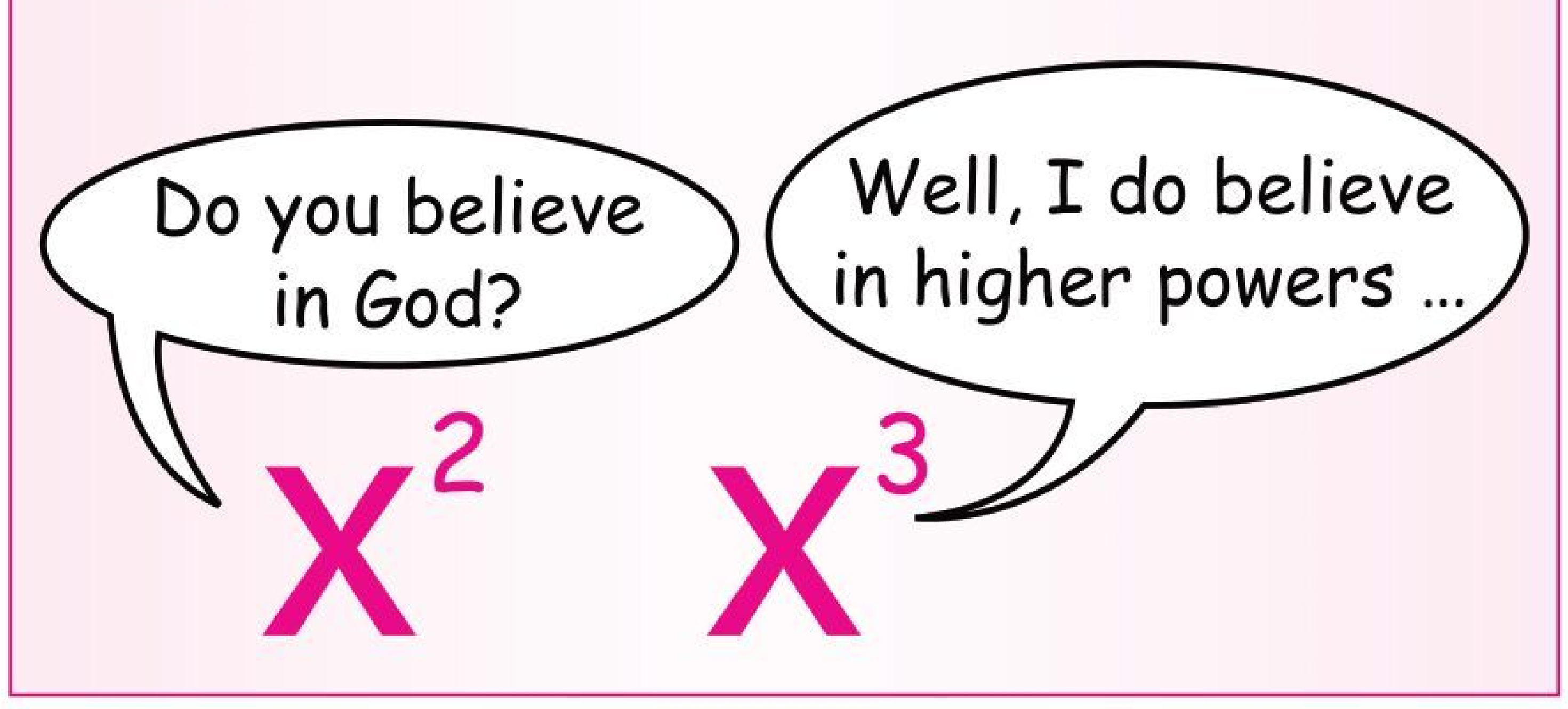
8. If $\cot^{-1}\left(\frac{n^2 - 10n + 21.6}{\pi}\right) > \frac{\pi}{6}, n \in N$, then n can be

(a) 3 (b) 2 (c) 4 (d) 8

9. $2\tan^{-1}(-2)$ is equal to

(a) $-\cos^{-1}\left(\frac{-3}{5}\right)$ (b) $-\pi + \cos^{-1}\frac{3}{5}$

COMIC CAPSULE


Do you believe in God?
Well, I do believe in higher powers ...
X^2
X^3

(c) $-\frac{\pi}{2}+\tan^{-1}\left(-\frac{3}{4}\right)$

(d) $-\pi+\cot^{-1}\left(-\frac{3}{4}\right)$

10. $\theta=\tan^{-1}(2\tan^2\theta)-\tan^{-1}\left\{\left(\frac{1}{3}\right)\tan\theta\right\}$, if

(a) $\tan\theta=-2$
(b) $\tan\theta=0$
(c) $\tan\theta=1$
(d) $\tan\theta=2$

11. If the numerical value of

$$\tan\left\{\tan^{-1}\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right\}\text{ is }\frac{a}{b},\text{ then}$$

(a) $a+b=23$
(b) $a-b=11$
(c) $3b=a+1$
(d) $2a=3b$

12. If $\sin^{-1}\left(a-\frac{a^2}{3}+\frac{a^3}{9}+....\right)+\cos^{-1}(1+b+b^2+....)$

$=\frac{\pi}{2}$, then

(a) $b=\frac{2a-3}{3a}$
(b) $b=\frac{3a-2}{2a}$
(c) $a=\frac{3}{2-3b}$
(d) $a=\frac{2}{3-2b}$

13. If $x>y>z>0$, then the value of

$$\left[\cot^{-1}\frac{xy+1}{x-y}+\cot^{-1}\frac{yz+1}{y-z}+\cot^{-1}\frac{zx+1}{z-x}\right],\text{ where}$$

$[\cdot]$ represents the greatest integer function, is

(a) 2
(b) 3
(c) 0
(d) 4

Comprehension Type

Let $a=\cos^{-1}\cos 20$, $b=\cos^{-1}\cos 30$ and $c=\sin^{-1}\sin(a+b)$, then

14. The largest integer x for which $\sin^{-1}(\sin x)\geq|x-(a+b+c)|$ is

(a) 1 (b) 2 (c) 3 (d) 4

15. If $5\sec^{-1}x+10\sin^{-1}y=10(a+b+c)$, then the value of $\tan^{-1}x+\cos^{-1}(y-1)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) π (d) 0

Matrix Match Type

16. Match the following :

	Column I		Column II
P.	If $(\sin^{-1}x)^2+(\sin^{-1}y)^2=\frac{\pi^2}{2}$ then $x^3+y^3=$	1.	1
Q.	If $(\cos^{-1}x)^2+(\cos^{-1}y)^2=2\pi^2$ then $x^5+y^5=$	2.	−2
R.	If $(\sin^{-1}x)^2(\cos^{-1}y)^2=\frac{\pi^4}{4}$, then $\lvert x-y\rvert=$	3.	0
S.	If $\lvert\sin^{-1}x-\sin^{-1}y\rvert=\pi$ then $x^y=$	4.	2

	P	Q	R	S
(a)	2, 3	2, 4	3, 1	1
(b)	2, 3, 4	2	3, 4	1
(c)	2, 3, 4	1	2, 3	3
(d)	2, 3, 4	3	2, 3	1

Numerical Answer Type

17. The number of real solutions of the equation

$$\frac{(\sin^{-1}x)^3+(\cos^{-1}x)^3}{(\tan^{-1}x+\cot^{-1}x)^3}=7\text{ is \_\_\_\_.}$$

18. If $x=\tan\left[\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right]$, then $29x$ is equal to ____.

19. If $\sin^{-1}x+\sin^{-1}y=\pi$ and $x=\lambda y$, then the value of $39^{2\lambda}+5^{\lambda}$ must be ____.

20. If $\cos^{-1}(x)+\cos^{-1}(y)+\cos^{-1}(z)=\pi[\sec^2(u)+\sec^4(v)+\sec^6(w)]$, where u, v and w are least non-negative angles such that $u<v<w$, then the value of

$$x^{2000}+y^{2002}+z^{2004}+\frac{36\pi}{u+v+w}\text{ is \_\_\_\_.}$$

⁂

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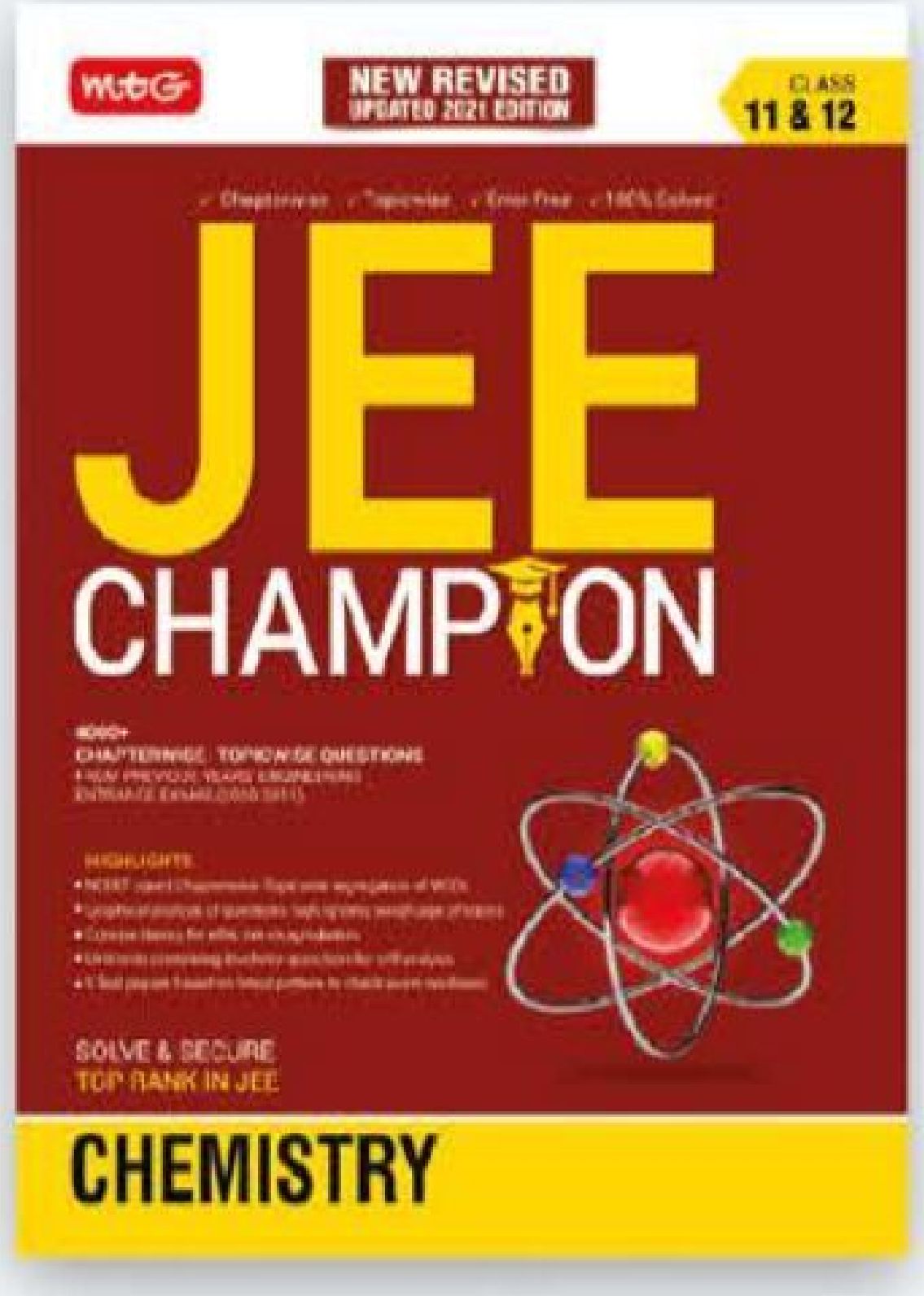
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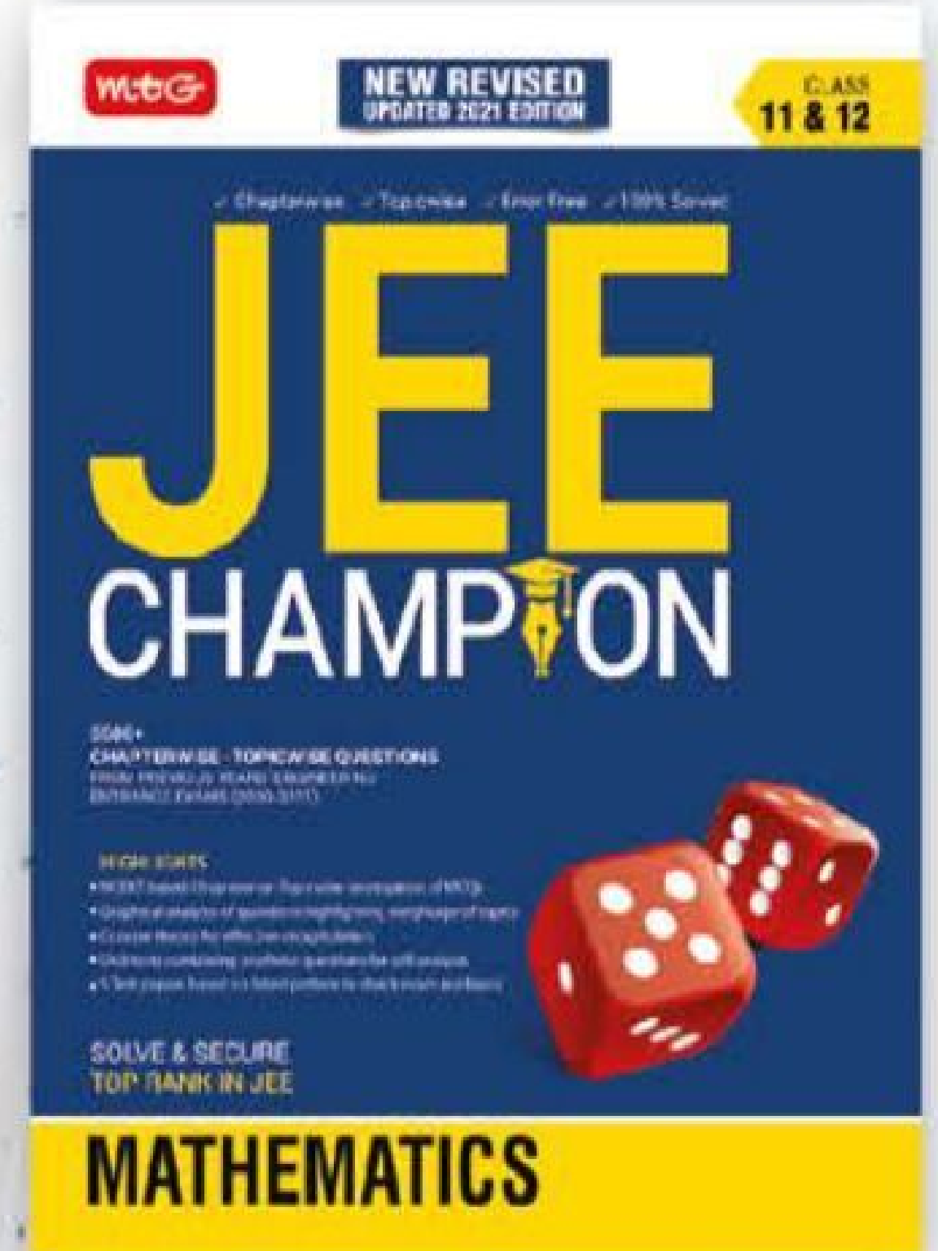
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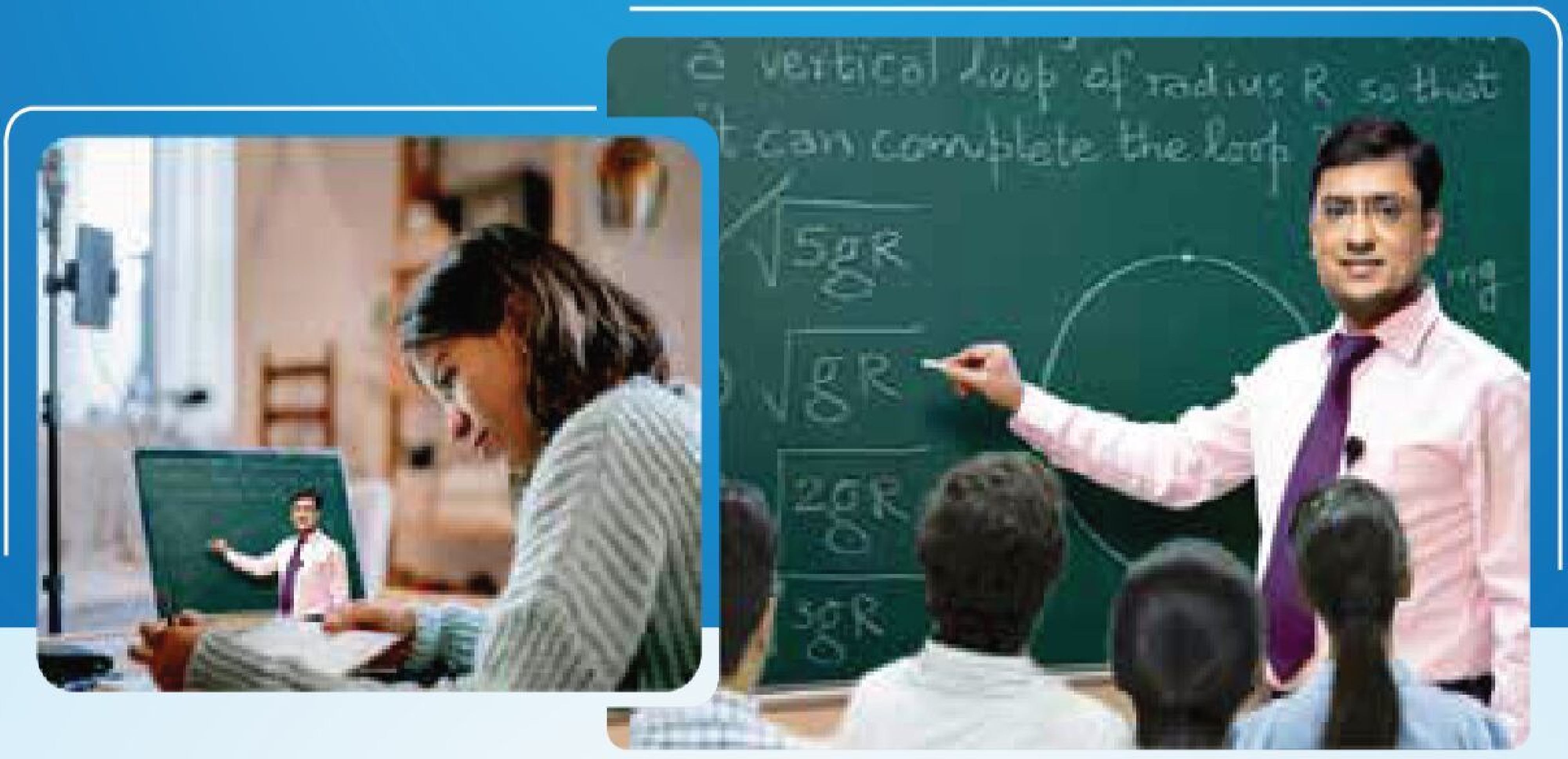

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